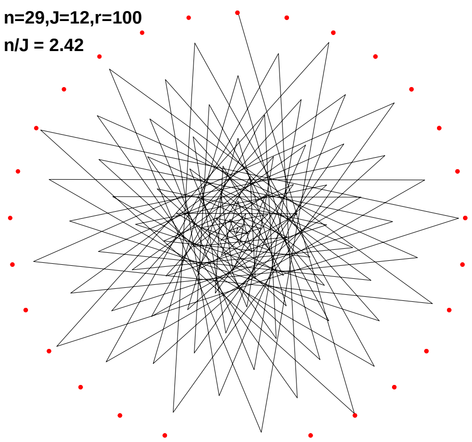
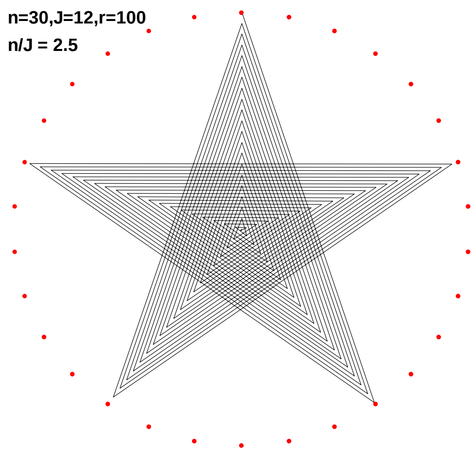
Swirls from Twisted Stars

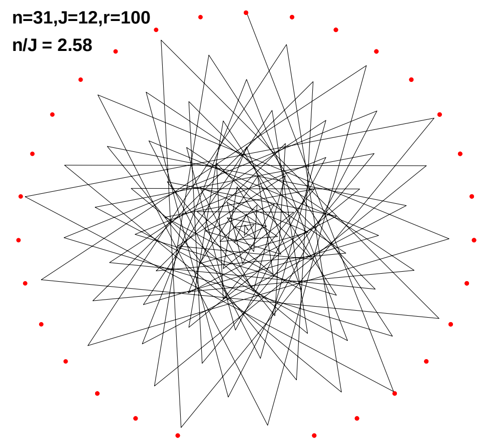
Swirls are related to just-over and just-under situations involving almost stars, just like they were with [almost polygons](). The difference here is that one cannot simply decrease or increase ***n*** by one to create the swirl. The difficulty is that stars involve multiple times around before the image is created, not one time around with polygons. For example, the simplest star, the 5,2-star or pentagram, is created by connecting vertices 0-2-4-1-3-0; the move from 4-1 involves beginning the second time around.

The problem with adjusting *n*. The first three images show the problem that arises by moving ***n*** one less or more than a value of ***n*** producing an *almost* 5,2-star for a given ***J*** (*almost* for the same reason we see almost polygons when ***n*** is a multiple of ***J***). An almost 5,2-star occurs when ***n/J*** = 5/2 = 2.5, just like the middle image. This image is an almost pentagram because each successive endpoint is 5/***r*** = 5/100 = 1/20 closer to the center than the previous point on the same vertex radius. If we decrease or increase ***n*** by 1 like shown at left and right, the curves in the center seem to be both clockwise and counterclockwise at the same time, creating flower-like images rather than swirls.

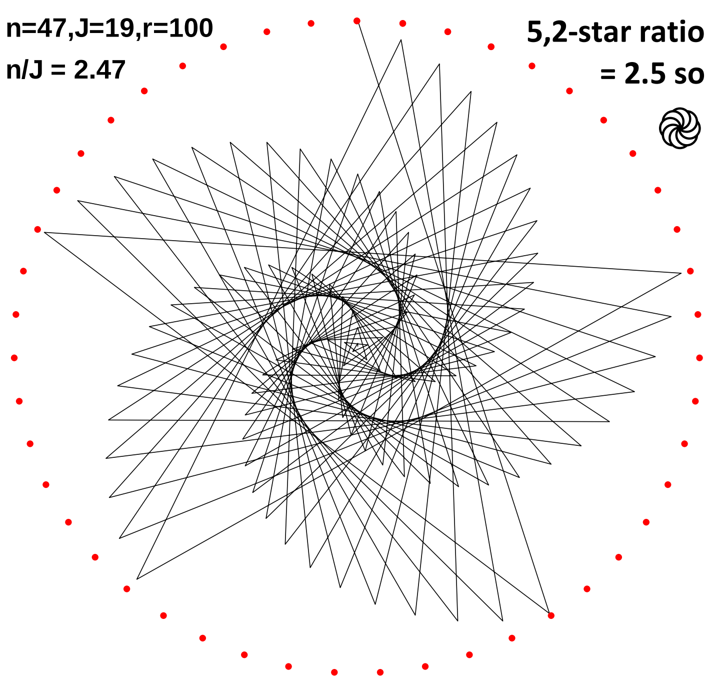
Focus on where the 5th endpoint is located. At left, it is 1/20 in from vertex 2 on the vertex 2 radius [**MA.** In modular terms: 5·12 = 60 = 2 mod 29]. At right, it is 1/20 in from vertex 29 on the vertex 29 radius (5·12 = 29 mod 31). [**MA.** Since 29 and 31 are prime, there are endpoints on all vertex radii in both images. At left, the vertex 28 radius is after 12 lines (12·12 = 144 = 28 mod 29) and the vertex 1 radius is after 17 lines (17·12 = 204 = 1 mod 29). At right, the vertex 1 radius occurs after 13 lines (13·12 = 156 = 1 mod 31) and the vertex 30 radius occurs after 18 lines (18·12 = 216 = 30 mod 31).]

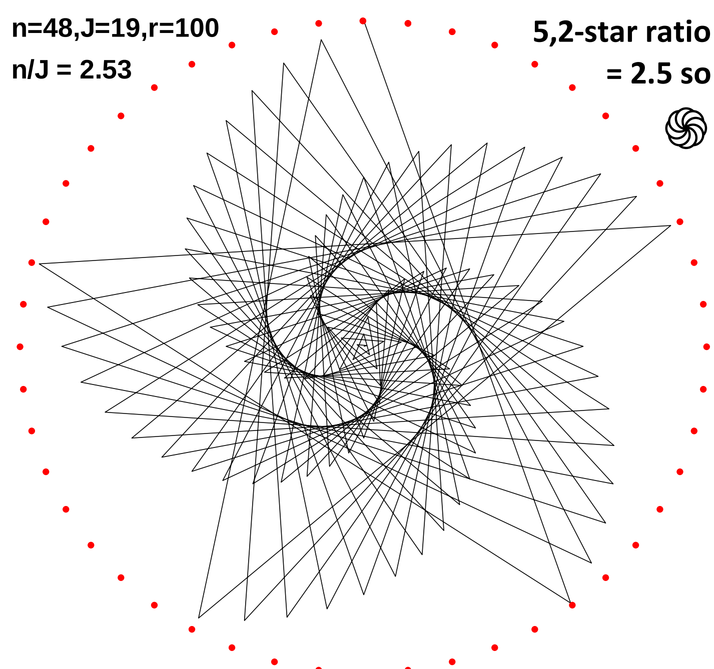






Focus on *n/J*. If ***n/J*** is a ratio close to 2.5, one obtains twisted pentagrams if ***n/J*** < 2.5 and pentagrams if ***n/J*** > 2.5.





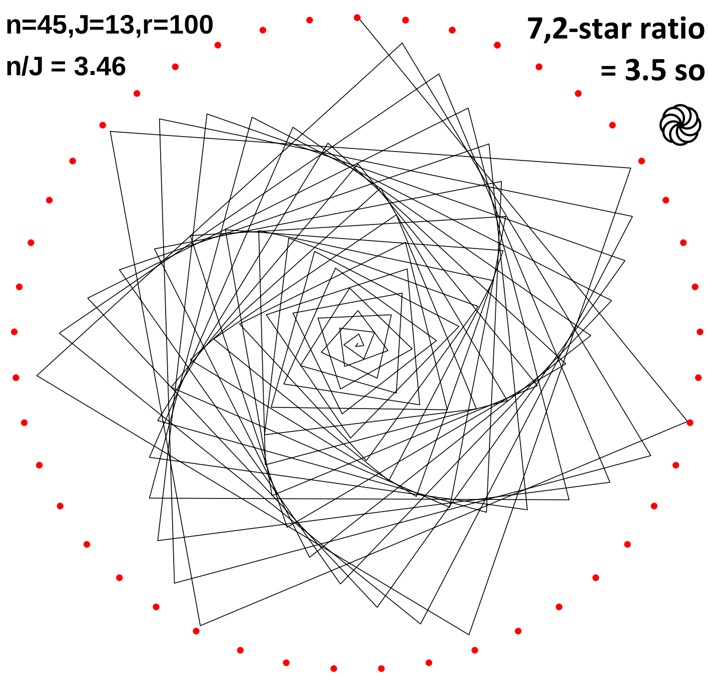
Notice that the 5th line ends on the vertex 1 radius in the left image and on the vertex ***n***-1 radius in the right image. The left image shows *overly-closed* pentagrams and the right shows *cracked-open* pentagrams. The swirls in these images can be seen both from the curves created by lines, and from the curves created by points, much as was described in E9.3.

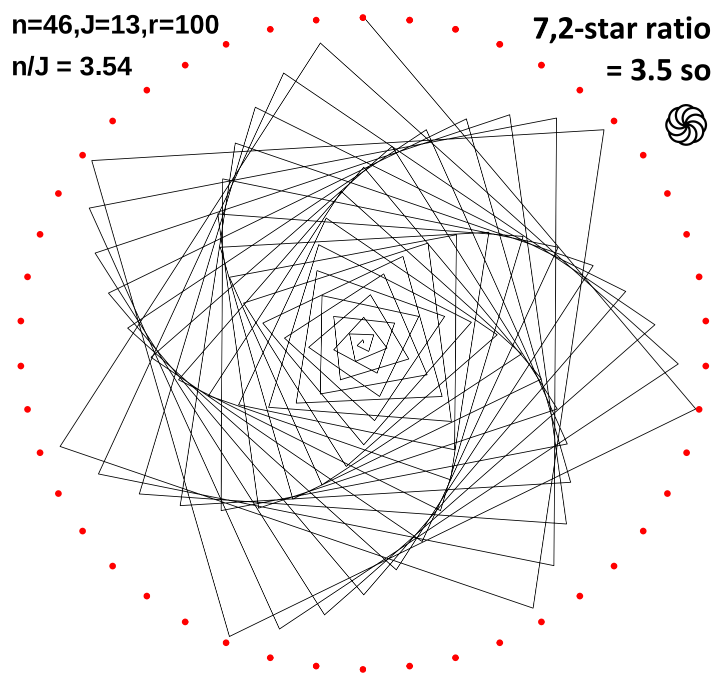
Beyond pentagrams. As discussed in E2.2.1, there are no continuously-drawn 6-point stars but there are two 7-point stars and one 8-point star. We begin by providing examples of swirling 7,2, 7,3, and 8,3-stars then examine larger point stars as well. In general, an ***a***,***b***-star with ***b*** < ***a***/2 will have ***a*** points if ***a*** and ***b*** are coprime, GCD(***a***,***b***) = 1. (We use ***a***,***b***-star rather than ***n***,***J***-star because ***n*** and ***J*** are values of the parent image.)

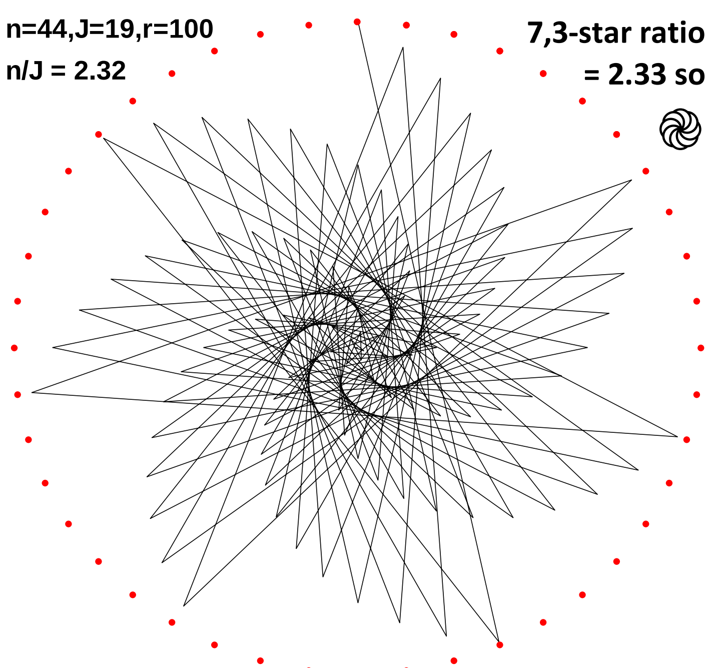
CLAIM. An ***a***,***b***-swirl occurs if ***n***/***J*** is close to ***a***/***b***, with ***n*** and ***J*** coprime, ***a*** and ***b*** coprime, and the end of the ***a***th line on a vertex radius that is within one vertex of the top.

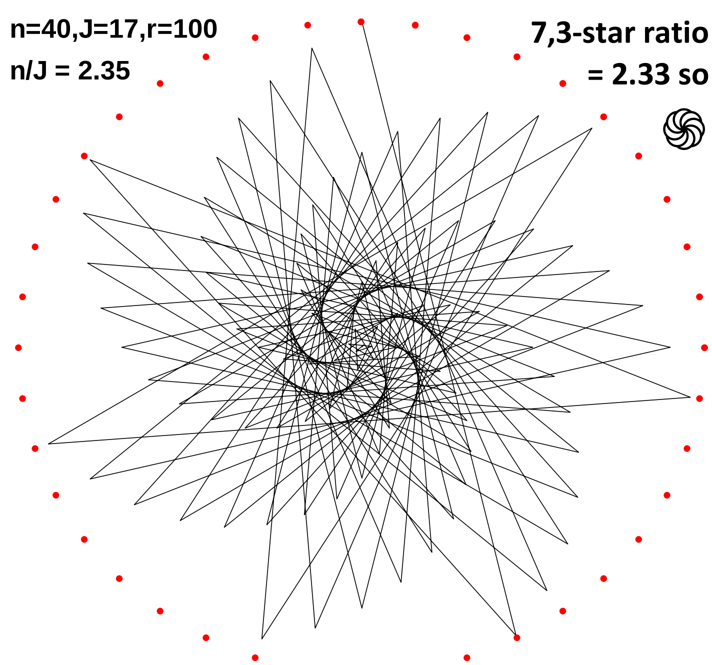
Left: If the ***a***th endpoint is on the vertex 1 radius, ***n***/***J*** < ***a***/***b***, and the image is .

Right: If the ***a***th endpoint is on the vertex ***n***-1 radius, ***n***/***J*** > ***a***/***b***, and the image is .



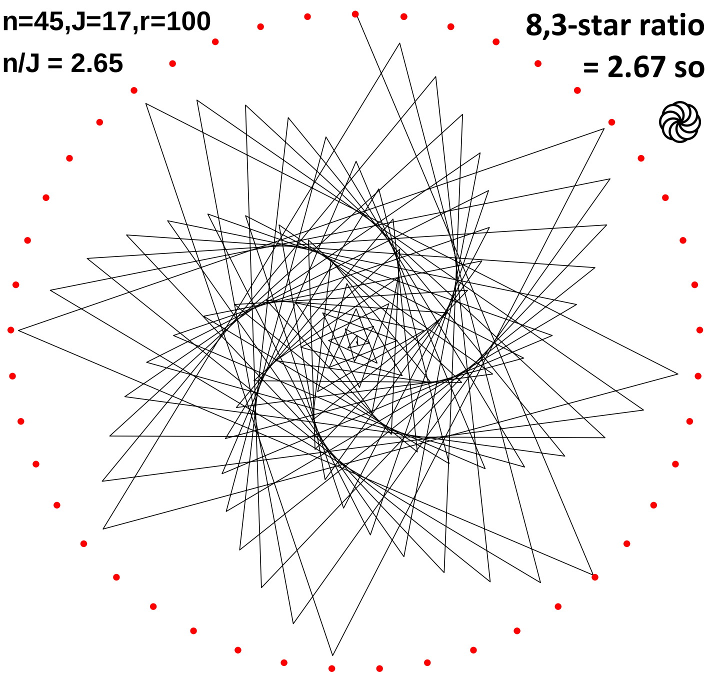


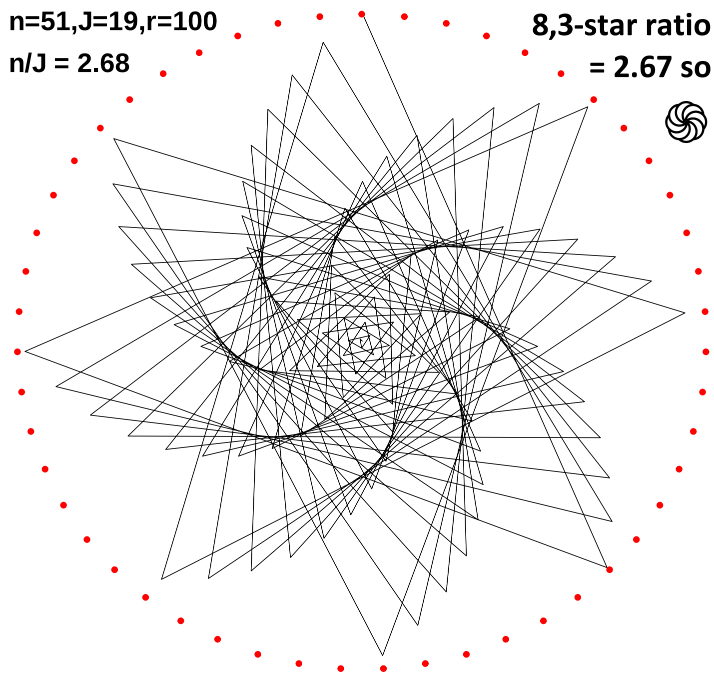




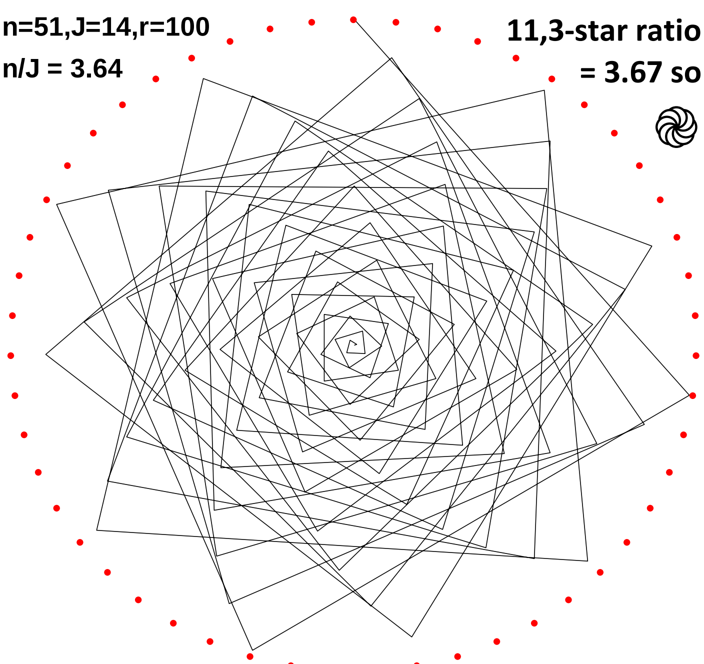
Comparing the ***a***,***b*** = 7,2- and 7,3- swirls, one sees that the smaller is ***b*** for given ***a***, the more the resulting image resembles *almost polygons* swirls. They are not the same, of course, but notice for example, that only the first 3 to 4 steps are not cross-hatched in the 7,2 version versus these ***n*** = 48, ***J*** = 7, ***r*** = 200 and ***n*** = 50, ***J*** = 7, ***r*** = 200 twisted 7-gon versions.

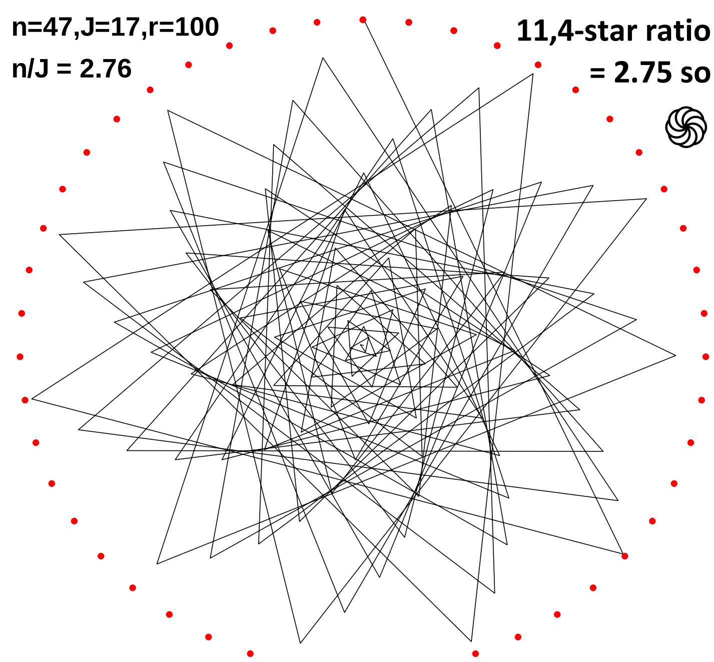
Notice how the "almost squares" in the middle of these 8,3-star swirls seem to contain the 8 swirls in both images.





Finally, two 11-point swirls are provided, one , the other, . Both are provided to show how easy it is to determine the ***a***,***b*** value for each image. To determine ***a*** and ***b***, simply count around the edge of the image.





One can find the number of points, ***a***, and number of vertex jumps, ***b***, in a larger ***a***,***b***-swirl using the *Single Lines Overlaid Drawing Mode* set on *Pause.* Advance the number of *Drawn Lines* until the last line ends within one vertex of the top. Try this yourself with this ***n*** = 53, ***J*** = 19, ***r*** = 200 image. By advancing the number of *Drawn Lines*, you can quickly determine that the final image is a 14,5 because the 14th line ends on the vertex 1 radius (in modular terms, $14 \cdot 19 = 1 \bmod 53$).