## The Same Subdivisions Dots are used to Create Irregular Single-Cycle 7-gons and 7-grams

As long as $\boldsymbol{S}$ is a multiple of 7 , the same subdivision endpoints are used regardless of shape. The images come in sets of 3 with the only difference being the order in which those subdivision endpoints are used. This can be most cleanly explained by superimposing three variations of a 7 -gram on a regular decagon, $\boldsymbol{n}=10, \boldsymbol{J}=1$, for $\boldsymbol{S}=7, \boldsymbol{P}=10 \boldsymbol{k}$ with $\boldsymbol{k}$ ranging from 1 to 6 . (Note: With $\boldsymbol{S}=7$ and $\boldsymbol{n}=10, \boldsymbol{P}=\operatorname{ROUND}(\boldsymbol{k n S} / 7,0)=10 k$.) There are 70 red subdivision dots, and each image uses only multiples of 10 (this makes counting very easy). The first used dot in the $\boldsymbol{k}=1$ version uses the third dot on the second VF line ( $10=7 \cdot 1+3$ ) while the first used dot in the $\boldsymbol{k}=2$ version uses the $6^{\text {th }}$ dot on the third VF line ( 20 $=7 \cdot 2+6$ ), and so on. You should be able to locate the first used subdivision point for $\boldsymbol{k}=3$ through 6 on the image below using the counting rule $\boldsymbol{P}=10 \boldsymbol{k}$. The table provides complete results for each $\boldsymbol{k}$.

The situation is shown with one final superimposition, the black dots are the vertices of a regular 7-gon labelled in red. This superimposition makes clear that none of the 7-grams are regular. In this instance, they are near the regular 7-gon vertices but none of the 7-gram vertices, except for the top vertex, are on the circle containing the regular 7-gon.

| End |  | $=k$ | Near |  | $=k$ | Near |  | $=k$ | Near |  | $=k$ | Near |  | $=k$ | Near |  | $=k$ | Near |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { of } \\ \text { line } \\ \# \\ \hline \end{gathered}$ | Total dots counted | Total MOD 70 | regular <br> 7-gon vertex | Total <br> dots counted | Total MOD 70 | $\begin{gathered} \text { regular } \\ 7 \text {-gon } \\ \text { vertex } \end{gathered}$ | Total dots counted | Total MOD 70 | regular <br> 7-gon <br> vertex | Total dots counted | Total MOD 70 | $\begin{gathered} \text { regular } \\ 7 \text {-gon } \\ \text { vertex } \end{gathered}$ | Total dots counted | Total <br> MOD 70 | regular <br> 7-gon <br> vertex | Total dots counted | Total MOD 70 | $\begin{gathered} \text { regular } \\ \text { 7-gon } \\ \text { vertex } \end{gathered}$ |
| 1 | 10 | 10 | 1 | 20 | 20 | 2 | 30 | 30 | 3 | 40 | 40 | 4 | 50 | 50 | 5 | 60 | 60 | 6 |
| 2 | 20 | 20 | 2 | 40 | 40 | 4 | 60 | 60 | 6 | 80 | 10 | 1 | 100 | 30 | 3 | 120 | 50 | 5 |
| 3 | 30 | 30 | 3 | 60 | 60 | 6 | 90 | 20 | 2 | 120 | 50 | 5 | 150 | 10 | 1 | 180 | 40 | 4 |
| 4 | 40 | 40 | 4 | 80 | 10 | 1 | 120 | 50 | 5 | 160 | 20 | 2 | 200 | 60 | 6 | 240 | 30 | 3 |
| 5 | 50 | 50 | 5 | 100 | 30 | 3 | 150 | 10 | 1 | 200 | 60 | 6 | 250 | 40 | 4 | 300 | 20 | 2 |
| 6 | 60 | 60 | 6 | 120 | 50 | 5 | 180 | 40 | 4 | 240 | 30 | 3 | 300 | 20 | 2 | 360 | 10 | 1 |
| 7 | 70 | 0 | 0 | 140 | 0 | 0 | 210 | 0 | 0 | 280 |  | 0 | 350 |  | 0 | 420 | 0 | 0 |
| Clockwise 7,1-star |  |  |  | Clockwise 7,2-star |  |  | Clockwise 7,3-star |  |  | Counter-clockwise 7,3-star |  |  | Counter-clockwise 7,2-star |  |  | Counter-clockwise 7,1-star |  |  |

One final thing to note is that each image is drawn twice, once clockwise the other time counter-clockwise. The clockwise versions are the smaller $\boldsymbol{k}$ values which makes sense because $\boldsymbol{J}=1$. Had we set $\boldsymbol{J}=\boldsymbol{9}=\boldsymbol{n}-1$, then all the images would be drawn in the opposite direction. As a result, we will restrict our analysis to $\boldsymbol{k}=1,2,3$ given a 7-step image.


Although these images are irregular, they do have the vertical symmetry that is common to all string art images.

We could have created different but similar images for other values of $\boldsymbol{n}$. Each would result in irregular 7-grams when $J$ is "small" relative to $n / 2$, but for larger $J$, more complex images emerge. Among those images is the one that gives rise to three shape-shifting triangles, an example of which is shown at bottom right.
(11,7,11,5)


