## The Same Subdivisions Dots are used to Create Irregular Single-Cycle 7-gons and 7-grams

As long as **S** is a multiple of 7, the same subdivision endpoints are used regardless of shape. The images come in sets of 3 with the only difference being the order in which those subdivision endpoints are used. This can be most cleanly explained by superimposing three variations of a 7-gram on a regular decagon,  $\mathbf{n} = 10$ ,  $\mathbf{J} = 1$ , for  $\mathbf{S} = 7$ ,  $\mathbf{P} = 10\mathbf{k}$  with  $\mathbf{k}$  ranging from 1 to 6. (Note: With  $\mathbf{S} = 7$  and  $\mathbf{n} = 10$ ,  $\mathbf{P} = \text{ROUND}(\mathbf{knS}/7,0) = 10\mathbf{k}$ .) There are 70 red subdivision dots, and each image uses only multiples of 10 (this makes counting very easy). The first used dot in the  $\mathbf{k} = 1$  version uses the third dot on the second VF line ( $10 = 7 \cdot 1 + 3$ ) while the first used dot in the  $\mathbf{k} = 2$  version uses the  $6^{\text{th}}$  dot on the third VF line ( $20 = 7 \cdot 2 + 6$ ), and so on. You should be able to locate the first used subdivision point for  $\mathbf{k} = 3$  through 6 on the image below using the counting rule  $\mathbf{P} = 10\mathbf{k}$ . The table provides complete results for each  $\mathbf{k}$ .

The situation is shown with one final superimposition, the black dots are the vertices of a regular 7-gon labelled in **red**. This superimposition makes clear that none of the 7-grams are regular. In this instance, they are near the regular 7-gon vertices but none of the 7-gram vertices, except for the top vertex, are on the circle containing the regular 7-gon.

End	1 = <b>k</b>		Near	2	= <b>k</b>	Near	3 = <b>k</b>		Near	4 = <b>k</b>		Near	5 = <b>k</b>		Near	6 = <b>k</b>		Near
of	Total		regular	Total		regular	Total		regular	Total		regular	Total		regular	Total		regular
line	dots	Total	7-gon	dots	Total	7-gon	dots	Total	7-gon	dots	Total	7-gon	dots	Total	7-gon	dots	Total	7-gon
#	counted	MOD 70	vertex	counted	MOD 70	vertex	counted	MOD 70	vertex	counted	MOD 70	vertex	counted	MOD 70	vertex	counted	MOD 70	vertex
1	10	10	1	20	20	2	30	30	3	40	40	4	50	50	5	60	60	6
2	20	20	2	40	40	4	60	60	6	80	10	1	100	30	3	120	50	5
3	30	30	3	60	60	6	90	20	2	120	50	5	150	10	1	180	40	4
4	40	40	4	80	10	1	120	50	5	160	20	2	200	60	6	240	30	3
5	50	50	5	100	30	3	150	10	1	200	60	6	250	40	4	300	20	2
6	60	60	6	120	50	5	180	40	4	240	30	3	300	20	2	360	10	1
7	70	0	0	140	0	0	210	0	0	280	0	0	350	0	0	420	0	0
	Clockwise 7,1-star			Clockwise 7,2-star			Clockwise 7,3-star			Counter-clockwise 7,3-star			Counter-clockwise 7,2-star			Counter-clockwise 7,1-star		

One final thing to note is that each image is drawn twice, once clockwise the other time counter-clockwise. The clockwise versions are the smaller k values which makes sense because J = 1. Had we set J = 9 = n-1, then all the images would be drawn in the opposite direction. As a result, we will restrict our analysis to k = 1, 2, 3 given a 7-step image.



Although these images are irregular, they do have the <u>vertical symmetry</u> that is common to all string art images.

We could have created different but similar images for other values of **n**. Each would result in irregular 7-grams when **J** is "small" relative to **n**/2, but for larger **J**, more complex images emerge. Among those images is the one that gives rise to <u>three shape-shifting</u> <u>triangles</u>, an example of which is shown at bottom right.

