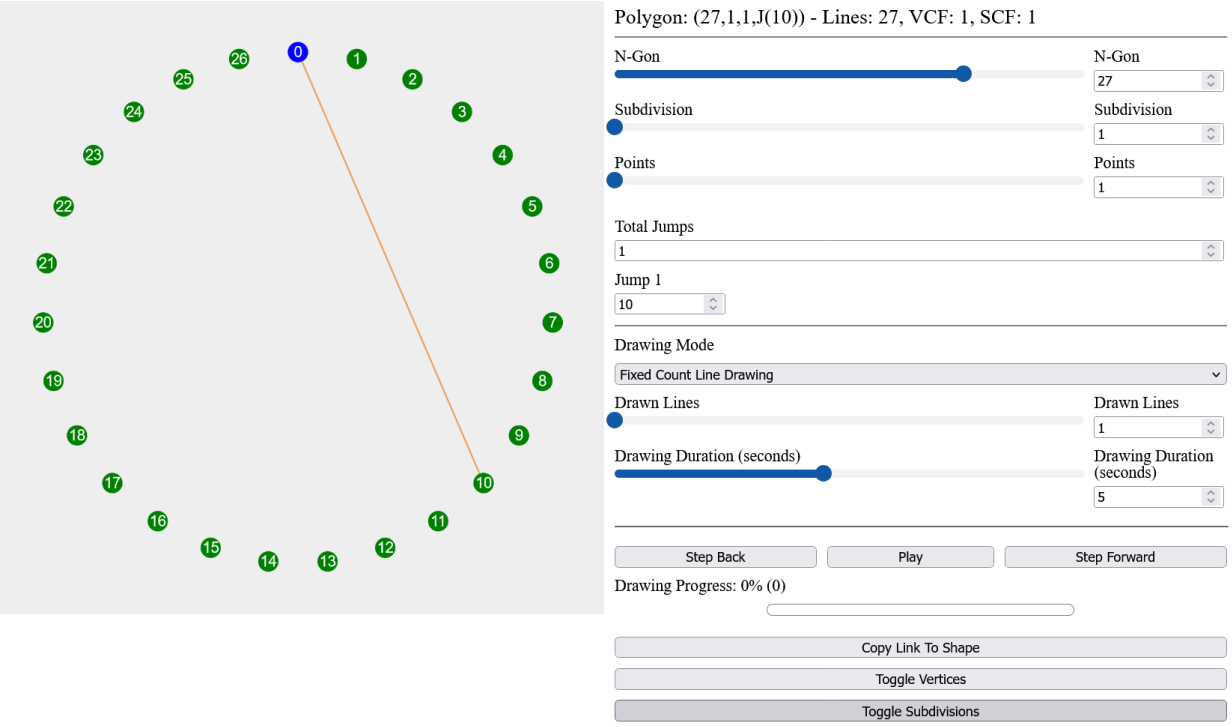
Using *n*,10-Stars to Visualize Last Digit Multiplication Patterns

Last Digit Patterns. One of the patterns that children notice as they learn multiplication has to do with the last digit in the answer. This is called the ones digit and there are 10 possible ones digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Some multiples use all 10 ones digits before repetition, and others do not. Compare, multiplication by 3 and 4.

> Multiplication by 3 uses all 10 ones digits in this sequence: 3, 6, 9, 2, 5, 8, 1, 4, 7, 0, …
>
> Multiplication by 4 uses only 5 ones digits in this sequence: 4, 8, 2, 6, 0, …

You can use ***n***,10-stars to show these patterns as an interactive exercise in a classroom that has just learned multiplication. (Conceptually, this material is connected to (but simpler than) E2.5.1, E4.2.2, and E5.2.)

Using *n*,*J*-stars to show last digit patterns. Use the *Home* mode of ***PwP*** and set ***S*** = ***P*** = 1, and ***J*** = 10. Choose any ***n*** > 20 so the 10th jump will be on the right half of the circle (27 is shown below). Click *Toggle Vertices* on, choose *Drawing Mode*, *Fixed Count Line Drawing*, click *Pause*, click to *No Drawing*, then return to *Fixed Count Line Drawing* and the first line will be shown, just like below (you need to go between modes because the default for each drawing mode is *Play*).



If you click *Play*, the vertices 1-9 (in **green** above) will get filled in in a specific order, based on ***n***. You can use the fill-in pattern of these digits to highlight the last digit multiplication pattern. Indeed, with ***n*** = 27, the pattern of vertex fill-in is the same as multiplication by 3 shown above. (*Drawing Duration* allows you to change how quickly vertices are filled in.)

A useful exercise is to stop the drawing once the image is close to completion (click *Pause* from the *Play*/*Pause* button), then use the *Step Forward* button to advance the image to completion, then *Step Forward* again. Change ***n*** and ask:

> ***How do you think the vertices will be filled in?***

You can provide a hint. Think of the tens and ones digit separately: $n = 10 \cdot n_2 + n_1$ (so that for $n = 27$, $n_2 = 2$ and $n_1 = 7$). Given this, the answer is simple. Vertices will be filled in in the same order as the multiplication by $10\text{-}n_1$. The reason is straightforward given how ***n***,10-stars are created: *A line is drawn every 10th vertex.* The first n_2 lines continue around the circle without going past the top. The n_2+1st line has n_1 vertices prior to the top, and the remaining $10\text{-}n_1$ vertices after the top before the next line is drawn (with our example, the 3rd line, is drawn from vertex 20 to 3 = 10-7). Therefore, to show multiplication by 4, use ***n*** = 26 or 36, but to show multiplication by 7, use ***n*** = 23 or 33, for example.

A final question: ***When will all vertices from 1-9 be uses in the final image?***

Answer: If ***n*** is not divisible by 2 or 5. Put another way, if $n = 10 \cdot n_2 + n_1$, then n_1 = 1, 3, 7, or 9.