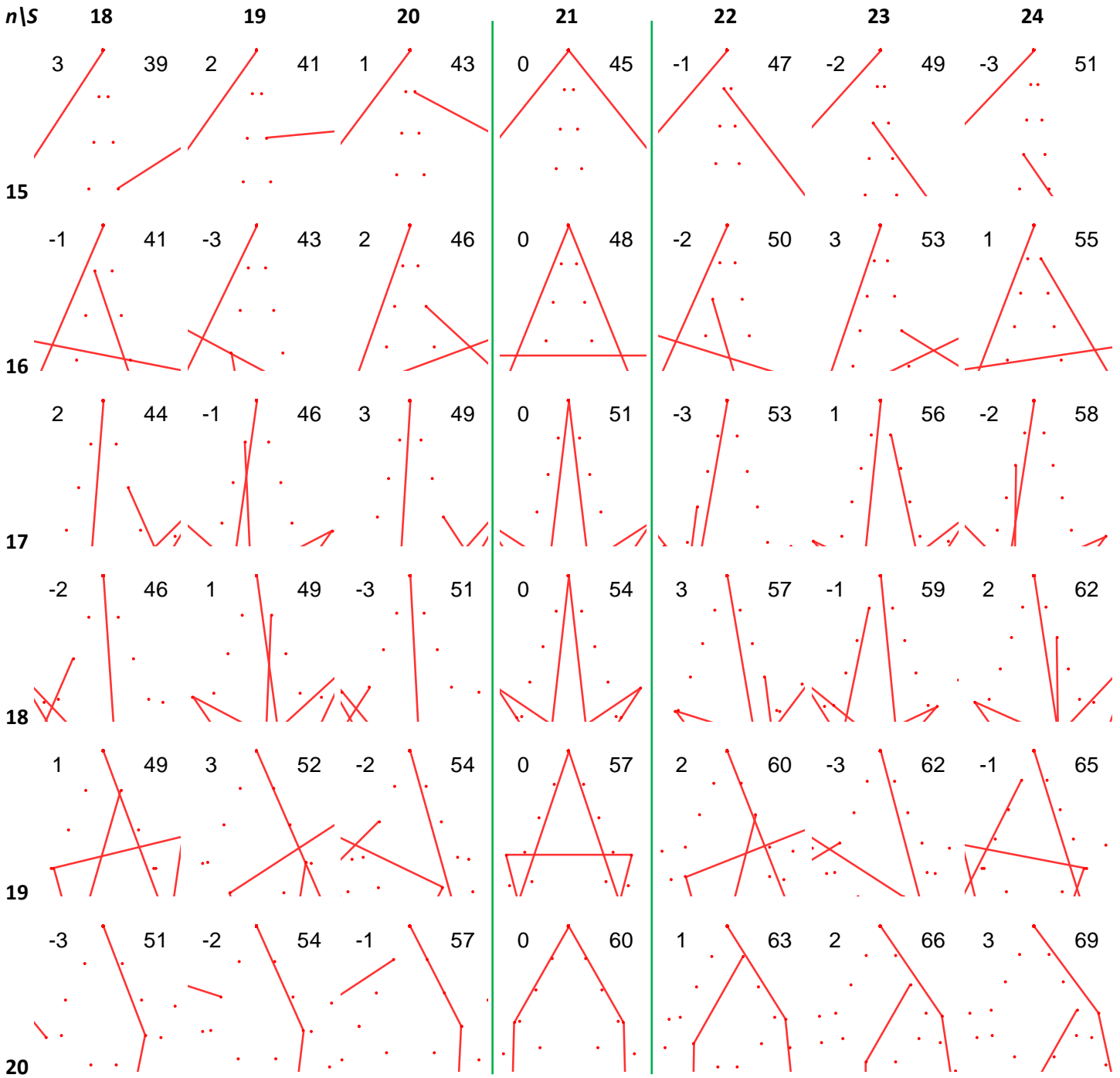# Variations on Small Images

This page shows 42 partial images (in a 7×6 array) which include subdivision dots, the start of the first line (starting at the top of the image), and the end of the 7th line. P uses *The 7-Line Generator Function*: P = ROUND($n·S$/7,0), so that the end of the 7th line is within 3 subdivisions of the top. P is noted to the right of the top in each image and the number to the left of the top of each image shows how *close to the top the 7th line Ends*, $E = 7P-n·S$. Each image has $J = 7$; n varies by row from 15 to 20 (since $n > 2J$, the first line of the vertex frame is slanted to the right, and hence so are the first subdivision endpoints). S varies by column from 18 to 24. S is centered on $S = 21$ so that $P = 3n$ with remainder 0 and these images are flanked by green vertical lines to orient the array.

n\S	18		19		20		21		22		23		24
15	3	39	2	41	1	43	0	45	-1	47	-2	49	-3	51
16	-1	41	-3	43	2	46	0	48	-2	50	3	53	1	55
17	2	44	-1	46	3	49	0	51	-3	53	1	56	-2	58
18	-2	46	1	49	-3	51	0	54	3	57	-1	59	2	62
19	1	49	3	52	-2	54	0	57	2	60	-3	62	-1	65
20	-3	51	-2	54	-1	57	0	60	1	63	2	66	3	69

As S deviates away from 21 we see spinning versions of the single-cycle image shown for $S = 21$ ($n = 15$ and 20 are 7-gons; $n = 16$ and 19 are 7,2-stars; and $n = 17$ and 18 are 7,3-stars). In each row there are two single-step images, one ending at subdivision 1 and the other at subdivision $n·S$-1 and these images are symmetric distances from $S = 21$.

How the patterns repeat. Note that increasing S by 1 adds n subdivision points and increasing n by 1 adds S subdivision points. This is the reason for the changes that occur in P and $E = 7P-n\cdot S$ as one parameter changes, holding the other parameter fixed. To make this explicit, compare E and P given n = 15 with n = 18, for the first and fourth row of images above.

n **= 15.** The total number of possible subdivisions $n\cdot S$ increases from 315 to 330 = 315+n as S increases from S = 21 to 22, because J = 7 so VCF = 1. P increases by 2 from 45 to 47 so 7P increases by 14, 1 less than the change in number of possible subdivisions or E changes from 0 to -1 given n = 2·7+1. Given this change in E of -1 for an increase by S of 1, one can understand why the pattern of E in the top row is 3, 2, 1, 0, -1, -2, -3 with P increasing by 2 for each change in S from 18 to 24 (to P = 51 when S = 24).

But if we increase to S = 25, the pattern of increasing P by 2 and decreasing E by 1 no longer holds. The change of -1 takes us to -4 = 3 mod 7. Therefore, P increases by 3 to 54 since 375 = 7·53+4 so 375/7 is closer to 54 than 53 and 7P is 3 more than 375 or E = 3. The final E pattern for n = 15 as S increases, starting from an S divisible by 7, is E = 0, -1, -2, -3, 3, 2, 1, 0, … .

n **= 18.** The total number of possible subdivisions $n\cdot S$ increases from 378 to 396 = 378+n as S increases from S = 21 to 22, because J = 7 so VCF = 1. P increases by 3 from 54 to 57 so 7P increases by 21, 3 more than the change in number of possible subdivisions or E changes from 0 to 3 given n = 3·7-3. A second increase in S to 23 gives 3+3 = 6 if P increases by 3, but -1 if P increases by 2. Therefore, P increases by 2 to 59 and E = -1 = 6 mod 7. A third increase to S = 24 adds 3 to -1 to obtain E = 2 with P = 62. Similar adjustments alternate between an increase in P of 2 or 3 in order to maintain the 7th endpoint between -3 ≤ E ≤ 3 of the top of the image. The final E pattern for n = 18 is E = 0, as S increases, starting from an S divisible by 7, is E = 0, 3, -1, 2, -2, 1, -3, 0, … .

These same patterns in E emerge as n changes for fixed S. The easiest to see is the first column where the number of subdivisions increases by S = 18 for each increase in n. The pattern is E = 3, -1, 2, -2, 1, -3, 0 as n increases from 15 to 21 because 3+3 = -1 mod 7, -1+3 = 2 mod 7, 2+3 = -2 mod 7, -2+3 = 1 mod 7, 1+3 = -3 mod 7 and -3+3 = 0 mod 7. The S = 22 column acts the same as the n = 15 row since 15 = 22 = 1 mod 7. The E pattern in that column is E = -1, -2, -3, 3, 2, 1, 0 as n increases from 15 to 21 by the same modular arithmetic subtracting 1 each time rather than adding 3.

Similar patterns could be obtained for changes in S for fixed n and changes in n for fixed S based on the remainder of the fixed parameter upon division by 7. To see the pattern, start from the mod 0 column (S = 21) and consider what happens as S increases to 22. For instance, the second row (n = 16) goes from E = -2 for S = 22 to E = 3 for S = 23, just like the start of the S = 23 column. Both have the same pattern because n = 16 and S = 23 each have remainder 2 when divided by 7.

Rounding Creates *Just-Under* and *Just-Over* Situations. An aspect of this analysis that is initially counterintuitive has to do with the sign of E. If the remainder is 1, 2, or 3 when $n\cdot S$ is divided by 7, then the ROUND function pulls P down a bit. That means that 7P will be *just under* $n\cdot S$ or E will be -1, -2, or -3. Conversely, if the remainder is 6, 5, or 4 when $n\cdot S$ is divided by 7, then the ROUND function pulls P up a bit. That means that 7P will be *just over* $n\cdot S$ or E will be 1, 2, or 3.

This information is summarized in the table below in four parts. The first two are shown to verify that one can use modular math to obtain closest values (note *the entries are the same* whether we look for the remainder of $n\cdot S$ (first part of the table) or the remainder of n mod 7 times S mod 7 (second part of the table). The third simply closes the remainder to the nearest multiple of 7 (by turning 4, 5, 6 into -3, -2, -1). The final shows E (the values in upper left of each partial image). **This SAME pattern exists for any set of n between multiples of 7, and centered on any S = 7·m.**

n between multiple of 7	Multiplication mod 7* S divisible by 7, S = 21 ||						Remainder multiplication mod 7** n	n mod 7	S divisible by 7, S = 21							Close: Subtracting 7 if Remainder > 3 n	n mod 7	S divisible by 7, S = 21							End of 7th line, E = -Close n	n mod 7	S divisible by 7, S = 21							S
	18	19	20	21	22	23	24	n	n mod 7	18	19	20	21	22	23	24	n	n mod 7	18	19	20	21	22	23	24	n	n mod 7	18	19	20	21	22	23	24	S mod 7
										4	5	6	0	1	2	3			4	5	6	0	1	2	3			4	5	6	0	1	2	3	
15	4	5	6	0	1	2	3	15	1	4	5	6	0	1	2	3	15	1	-3	-2	-1	0	1	2	3	15	1	3	2	1	0	-1	-2	-3	
16	1	3	5	0	2	4	6	16	2	1	3	5	0	2	4	6	16	2	1	3	-2	0	2	-3	-1	16	2	-1	-3	2	0	-2	3	1	
17	5	1	4	0	3	6	2	17	3	5	1	4	0	3	6	2	17	3	-2	1	-3	0	3	-1	2	17	3	2	-1	3	0	-3	1	-2	
18	2	6	3	0	4	1	5	18	4	2	6	3	0	4	1	5	18	4	2	-1	3	0	-3	1	-2	18	4	-2	1	-3	0	3	-1	2	
19	6	4	2	0	5	3	1	19	5	6	4	2	0	5	3	1	19	5	-1	-3	2	0	-2	3	1	19	5	1	3	-2	0	2	-3	-1	
20	3	2	1	0	6	5	4	20	6	3	2	1	0	6	5	4	20	6	3	2	1	0	-1	-2	-3	20	6	-3	-2	-1	0	1	2	3	
	*Each cell is =MOD($n\cdot S$,7)							**Each cell is =MOD(MOD(n,7)*MOD(S,7),7)																			E > 0, start of VF; E < 0, end of VF									