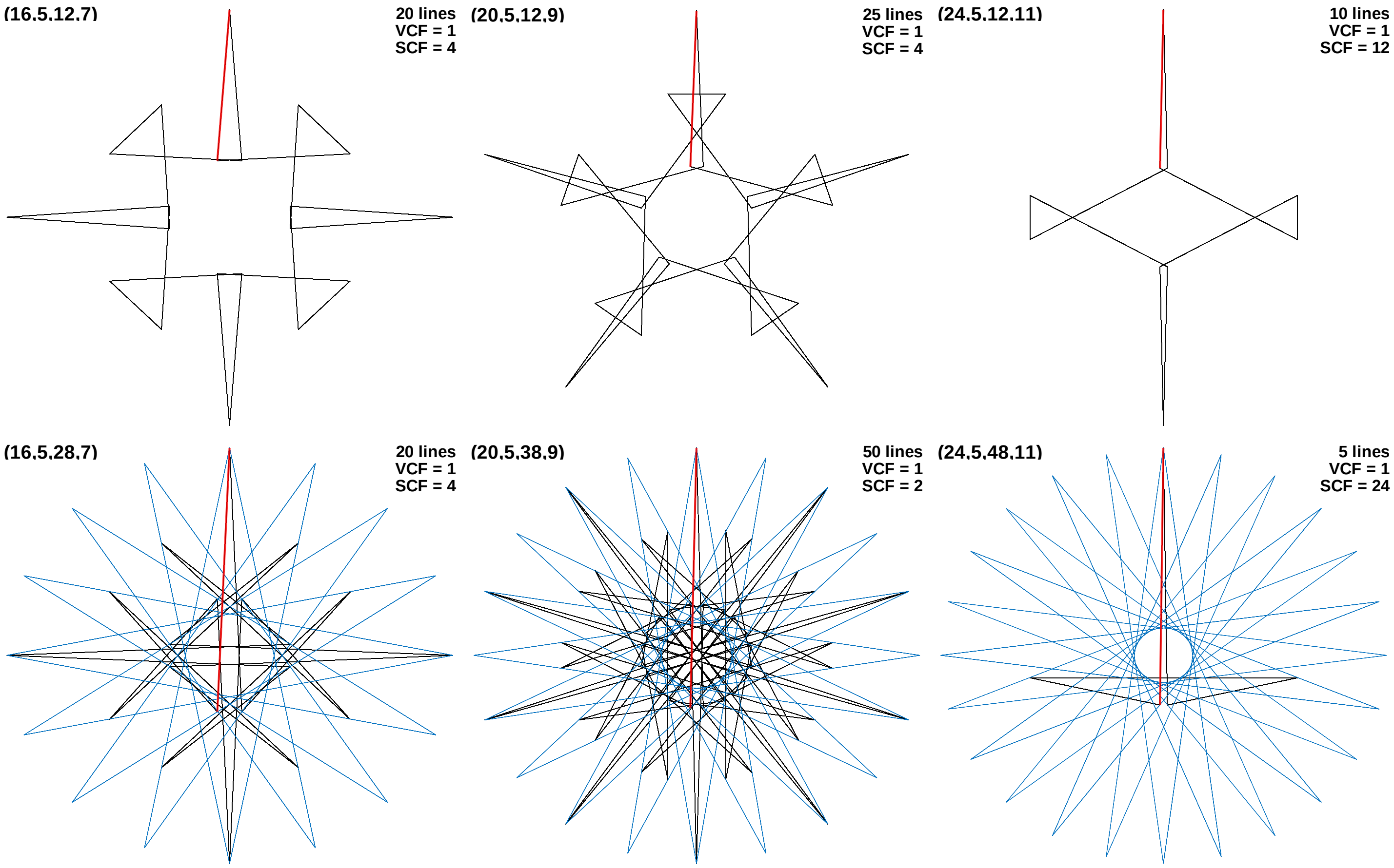
An Analysis of S = 5 given n = 4k, J = 2k-1 Sharpest Needles Images



The six images above show three consecutive ***n*** = 4***k***, ***J*** = 2***k***-1, ***S*** = 5 sharpest central needles images. The top row uses the pair of subdivision dots (not shown to reduce clutter) above the center and hence create images that avoid the donut hole, and the second row uses the pair below the center and create images that fill in the donut hole, E10.2.1. The number of lines varies greatly due to SCF varying between 2 and 24. In both rows, the **first line** (shown in **red**) is to the left of the vertical diameter and the needles get sharper as ***n*** increases; both attributes continue to be the case. The bottom row includes the **vertex frame in blue**, but this has been removed in the top row since the pattern is easier there.

Avoids the Donut Hole* Rule.** The first subdivision endpoint used is the second dot on the third line of the VF. Each line of the VF involves 5 subdivisions, so ***P = 12 = 2·5+2. The third line on the VF is from vertex ***n***-2, two to the left of the top, to vertex ***n***/2-3, three to the right of the bottom. This is a subdivision endpoint just to the left of the centerline above the center. The resulting image avoids the donut hole.

Fill in the Donut Hole* Rule.** The first subdivision endpoint used is the third dot on the VF line from vertex 3, three to the right of the top, to vertex ***n/2+2, two to the left of the bottom. This is a subdivision endpoint just to the left of centerline below the center. Lines in this image will often cross the *donut hole*. Unlike above, ***P*** depends on ***n*** = 4***k***. In this instance, ***P*** = 10***k***-12. Each increase in ***k*** increases the number of lines needed by 2 (10 subdivisions) to reach vertex 3 on the VF.

Why both Rules Work. When ***k*** > 3, the third vertex is properly in the 1st quadrant and the third vertex from the bottom is properly in the 4th quadrant. Consider the ***x*** component of the two endpoints in both instances. In terms of deviations from the centerline, vertex 1, ***n***-1, ***n***/2-1 and ***n***/2+1 all have the same magnitude ***x*** component, call this x_1 (***n***-1 and ***n***/2+1 are negative). Similarly, call the difference between first and second from the centerline x_2 and the difference between 2nd and 3rd, x_3. For reasons similar to those discussed in the odd case, as ***n*** increases, these distances approach one another but in terms of magnitude, $x_1 > x_2 > x_3$. The total ***x*** distance, **X**, between vertices ***n***-2 and ***n***/2-3 is the same as between vertices 3 and ***n***/2+2 and equals: $\mathbf{X} = 2x_1 + 2x_2 + x_3$. Due to the inequalities above, $2/5\mathbf{X} < x_1 + x_2$ so that the ***x*** coordinate of the point 2/5 of the way from ***n***-2 to ***n***/2-3 is negative. Similarly, $3/5\mathbf{X} > x_1 + x_2 + x_3$ so that the ***x*** coordinate of the point 3/5 of the way from 3 to ***n***/2+2 is negative. As we saw 4/7 = 0.571 < 0.6, so first line has ***x*** > 0 for (20,7,53,9).