

## An Introduction to Even Sharpest Central Needles Stars

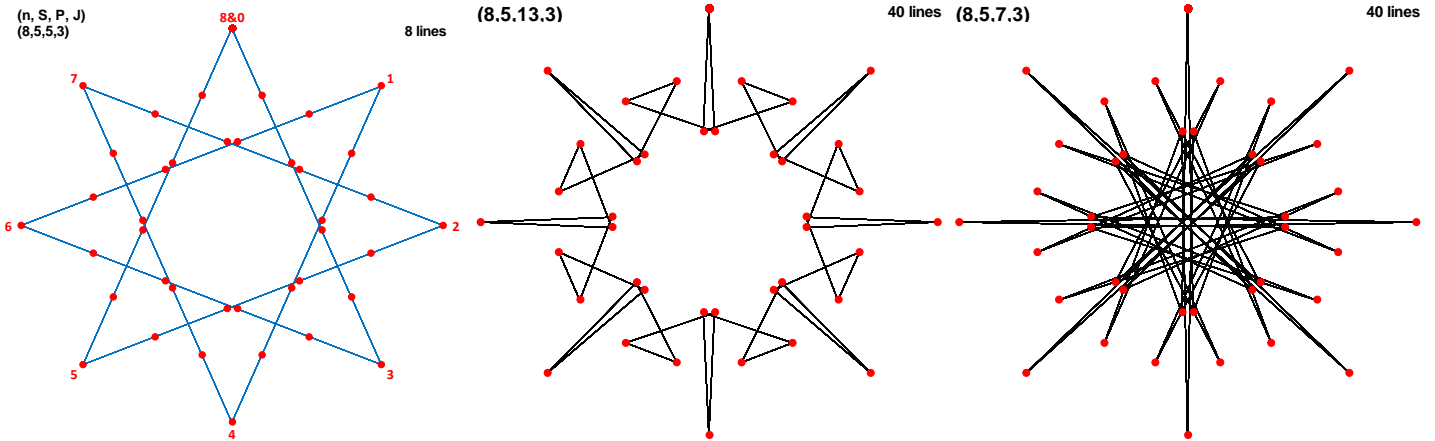
One of the simplest image patterns to explain is an odd needle star, examined in the [second ESA video](#), and [E11.7](#). These images are based on an odd  $n = 2k+1$ ,  $J = k$ ,  $S = 3$  and  $P = 4$ . See why needles get sharper as  $n$  increases [here](#).

An open question, posed at the end of E11.7 was, can you find examples of even needle stars, and can you obtain a simple rule governing their production like the rule provided above for odd  $n$ . Before examining this issue, it is worthwhile to explain more explicitly what we mean by a sharpest central needle.

**Sharpest Central Needles.** One can find images that have central needles that are sharper than the VF star by clicking [Vertex Frame](#) and [Subdivision Dots](#) on in the *Excel* file. The left 8,3-star VF shows **12 dots** that are inside the [VF angle between vertices 3-0-5](#) given  $S = 5$ . Of those, two pairs of dots are closer to the center line than others and if the first line is to one of these two points, and these pairs will have the sharpest needles possible given  $n$ ,  $S$ , and  $J$ . Given an even  $n$ , the two pairs are the same distance from one another. The middle image uses one of subdivisions in the top pair, [\(8,5,13,3\)](#) and is a *sharpest central needles* image that excludes the donut hole as discussed in [E10.2.1](#). The right shows the bottom pair, [\(8,5,7,3\)](#) is a sharpest central needles image that includes the donut hole. Both are versions of sharpest central needle images.

Were you to extend the middle lines for the vertical needle at 0, it would include the dots at the bottom. We can see this in another way, the vertical needle at 0 is inside the top pair of subdivisions. Additionally, it is visually clear that the angle created is smaller at right than at middle. This is the sharpest central needle because no subdivision dots (except for the opposite vertex) is in the cone created by the left and right half of the needle.

The word *central* is used to denote that the needles surround the center of the overall image (implicitly in the middle image and explicitly in the right). By contrast, the 2<sup>nd</sup> and 3<sup>rd</sup> line, and the 3<sup>rd</sup> and 4<sup>th</sup> line create two sharp needles that are not central. Both are oriented from SW to NE with points between 0-1 and 4-5 and both are to the SE of the center.



Additional issues when  $VCF > 1$  or  $SCF > 1$ . If either is larger than 1, fewer than  $n$  needles arise as we see below.  $VCF > 1$  means the image is simplified by dividing  $n$  and  $J$  by  $VCF$  (see [E6.1](#)), so the left image is identical to (7,5,12,3). But it is not always possible to simplify  $SCF > 1$  issues as we see at middle and right, both of which have  $n = 14$  and  $SCF > 1$ .

