Conditions that Guarantee a Jump Set Mirror has a Line of Symmetry

When there is a jump set, the web spiral mirror will not, in general, have a line of symmetry, LoS, as noted here. But there are jump set images in which the web mirror will have a LoS. Here we examine when that will happen.

Assume there are $\boldsymbol{k}$ jumps in the jump set. There are two conditions that guarantee a web mirror has a LoS.

1. The jumps need to be symmetric about $\boldsymbol{k}/2$ whether $\boldsymbol{k}$ is even or odd. $J_1 = J_k$, $J_2 = J_{k-1}$, … . If $\boldsymbol{k}$ is odd, there must be symmetry about the middle jump (the $(\boldsymbol{k}+1)/2^{nd}$ jump) and if $\boldsymbol{k}$ is even there must be two equal middle jumps (the $\boldsymbol{k}/2^{nd}$ and $\boldsymbol{k}/2+1^{st}$ jumps).
2. Twice the reduction factor, $\boldsymbol{r}$, must be a multiple of $\boldsymbol{k}$.

Rationale. For a LoS, one needs to have equal jump treatment at the start and at the end of the process, as well as everywhere in between. The start of the first line and the end of the last are the only two points at a radius of 1 from the center. The end of first (start of the second) and start of last (end of second to last) are the only two lines at a radial distance of $1\text{-}1/\boldsymbol{r}$, and so on. These start and end points of the first and last line must be J_1 jumps away from one another. A similar argument occurs for the second, and second to last lines, and so on.

To have overall jump set symmetry, the $2\boldsymbol{r}$ jumps involved in creating the final image must be symmetric about $\boldsymbol{r}$. This means that the $2\boldsymbol{r}^{th}$ jump must be the same as the first jump, and 2^{nd} jump is the same as the 2^{nd} to last jump and so on. This will happen if the $\boldsymbol{r}^{th}$ jump is either at the end of a jump set or in the middle of a jump set (since jump sets are themselves symmetric).

When it exists, where is the LoS? The LoS in the jump set case is like the LoS with a single jump if we use the same idea set out here for single jump models. Condition 2 implies that $2\cdot\boldsymbol{r}/\boldsymbol{k}$ is a whole number This is the number of jump sets in the mirrored image. The sum of jumps in a jump set is **Sum** = $J_1+J_2+\ldots+J_k$. Therefore, the sum of vertex jumps in the image (the equivalent of **F** in the single jump model) is the final vertex, **E** = MOD($2\cdot\boldsymbol{r}/\boldsymbol{k}\cdot$**Sum**, $\boldsymbol{n}$). The end of the $2\boldsymbol{r}^{th}$ line is opposite this vertex at $(\boldsymbol{E}+\boldsymbol{n}/2)$ and the LoS is the line connecting the center with the point on the unit circle associated with $(\boldsymbol{E}+\boldsymbol{n}/2)/2 = \boldsymbol{E}/2+\boldsymbol{n}/4$ which need not be a vertex of the $\boldsymbol{n}$-gon.

Small *r* symmetry. When $\boldsymbol{r} < \boldsymbol{k}$, it is possible to have a LoS even though $2\boldsymbol{r}$ is not a multiple of $\boldsymbol{k}$.

When $\boldsymbol{r} = 1$, the first jump is to the center and the second is all of the way out to the unit circle. The result is a V (created from a radius and an inverted radius), a vertical radius or a vertical diameter.

When $\boldsymbol{r} = 2$ and $\boldsymbol{k} = 3$, the second jump is to the center then the third jump ends the jump set but the fourth is the same as the third since it is the start of the second set. The same is not true for $\boldsymbol{r} = 2$ and $\boldsymbol{k} = 5$ as we see here.

Additionally, when $\boldsymbol{r} = 2$ and $\boldsymbol{k} = 4$, the second jump and third jump need not be the same. The same reasoning applies, the second jump is a jump into the center and this looks the same whatever the size of the second jump. This does not apply when $\boldsymbol{r}$ is a larger value that is divisible by 2 but not 4 like this $\boldsymbol{r} = 6$ example. In this instance, the start and end are at vertex 0 but the image has no LoS. Change this to $\boldsymbol{r} = 2$ and the image has a vertical LoS.