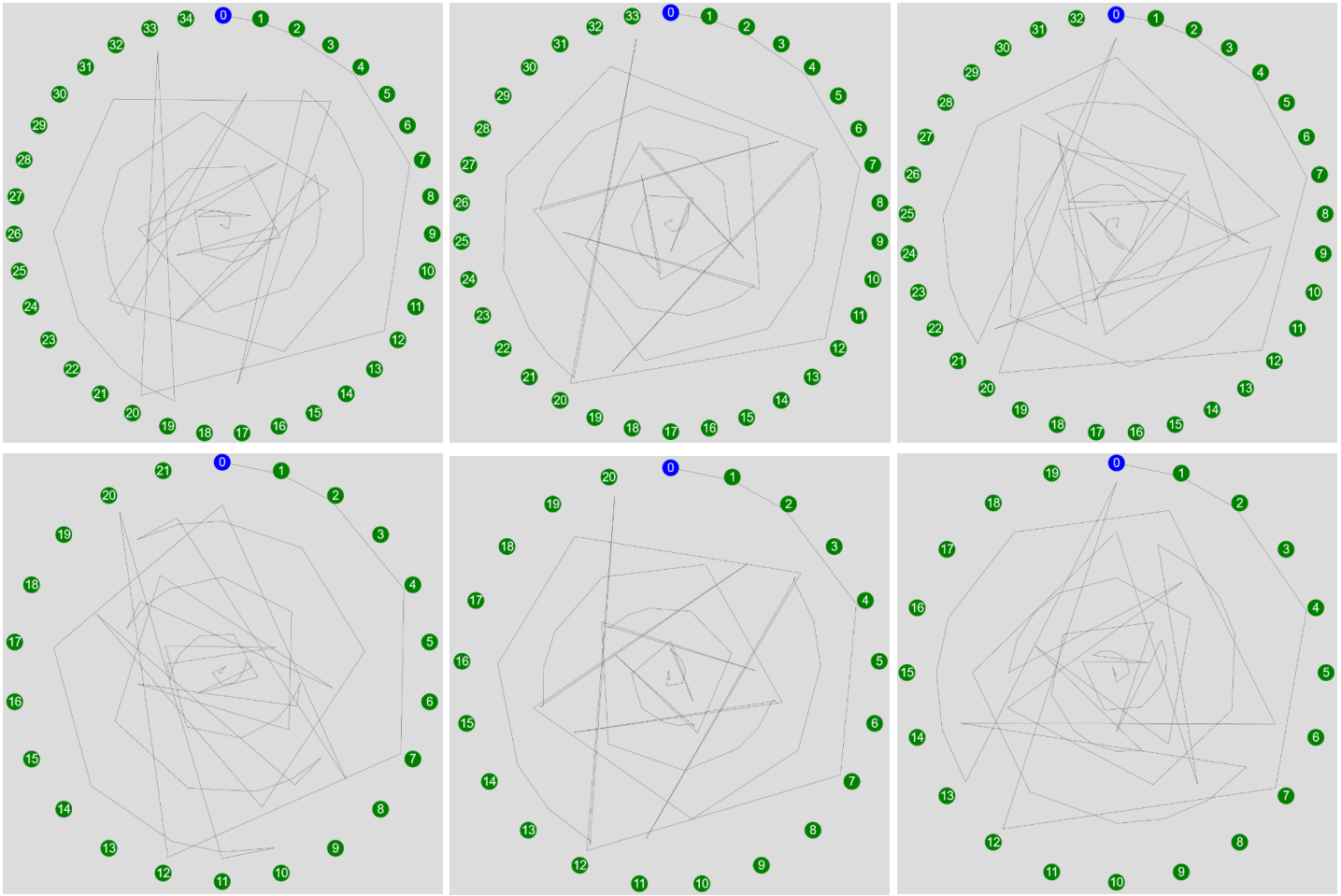
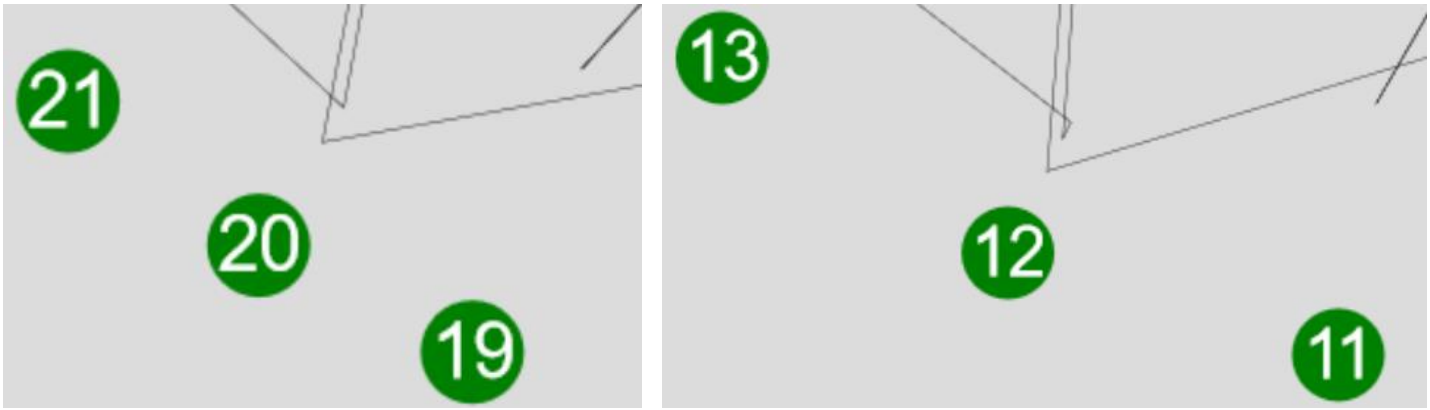
Internal Needles when Scrolling Across *n* with Fibonacci Spirals

Compare the two sets of three images based on the 8 jump set Fibonacci sequence (***n***,64,J(1,1,2,3,5,8,13,21)). Each decreases ***n*** by 1 from left to right and, in each case, the middle image seems qualitatively different from its left and right cousins. The first row shows ***n*** = 35, 34, 33 and second shows ***n*** = 22, 21, 20 from left to right (the link above is to ***n*** = 21).

The two middle images have sharp internal needles of various sizes that are spread throughout the image. In both middle images, the tip of the largest needle is at vertex ***n***-1. Beneath the six images are blow-ups of the eye of the largest needle at vertex radius 20 (left from upper middle) and 12 (right from lower middle).





Each image has 64 lines. Given ***r*** = 64, there are 64 lines to the center for each image. Of course, the lines near the center are difficult to count but in all, each **should** feel like it has the same number of lines. The middle images appear to have fewer lines, and this is due to the needles.

How the spiral needles are constructed. Spiral needles occur by having a pair of vertex jumps which sum to ***n***, or a multiple of ***n***. That way, the start and end of the base of the needle (where the eye of the needle would be) are on the same vertex radius separated by 2/***r*** (which is a small distance given ***r*** = 64). The sharpness of needles and the number of needles increases as ***r*** increases.

First row. The sum of the first six Fibonacci numbers is 20, so the starting point of line 7 is 6/64th in on the vertex 20 radius and the end of the 7th line is 7/64 in on the vertex 33 radius in all three first row images (recall that vertex 33 is the same as vertex 0 in the right image since ***n*** = 33). The 7th and 8th jumps in the jump set are 13 and 21, which sum to 34.

Left: Given ***n*** = 35, the end of the 8th jump is 8/64th in on the vertex 19 radius.

Middle: Given ***n*** = 34, the end of the 8th jump is 8/64th in on the vertex 20 radius. You can see this in the left blow-up as the two angles on the vertex 20 radius. The outer of the two angles is the end of 6th (from 12) and the start of 7th (to 33). The inner of the two angles is the end of the 8th (from 33) and the start of the next jump set (to 9/64th in on the vertex 21 radius).

Right: Given ***n*** = 33, the end of the 8th jump is 8/64th in on the vertex 21 radius.

Second row. The sum of the first five Fibonacci numbers is 12, so the starting point of line 6 is 5/64th in on the vertex 12 radius and the end of the 6th line is 6/64 in on the vertex 20 radius in all three second row images (recall that vertex 20 is the same as vertex 0 in the right image since ***n*** = 20). The 6th and 7th jumps in the jump set are 8 and 13, which sum to 21.

Left: Given ***n*** = 22, the end of the 7th jump is 7/64th in on the vertex 11 radius.

Middle: Given ***n*** = 21, the end of the 7th jump is 7/64th in on the vertex 12 radius. You can see this in the right blow-up as the two angles on the vertex 20 radius. The outer of the two angles are the end of 5th (from 7) and start of 6th (to 20). The inner of the two angles is the end of the 7th (from 20) and the start of the 8th jump. Since the 8th jump is 21 and ***n*** = 21, the end of this line is 8/64th in on the vertex 12 radius. The start of the 9th line is the start of the next jump set going to 9/64th in on the vertex 13 radius. *This is why there are three points in close succession on the vertex 12 radius in the blow-up (at 5/64, 7,64, and 8/64 in on the vertex 12 radius).*

Right: Given ***n*** = 20, the end of the 7th jump is 7/64th in on the vertex 13 radius.

The Second Largest Needle Tip. In the upper right corner of both blow-ups are the tips of the second largest needles.

First row middle image and left blowup. This tip is the end of the 15th line (7th line in second iteration of the jump set) at 15/64 in on the vertex 19 radius. [If you are wondering why the 7th point on the second jump set is vertex 19, it is because one jump set is 54 jumps and 7 jumps is 33, so 19 = 54+33 mod 34.]

Second row middle image and right blowup. This tip is the end of the 14th line (6th line in second iteration of the jump set) at 14/64 in on the vertex 11 radius. [The 6th point on the second jump set is vertex 11 because 6 jumps is 20, so 11 = 54+20 mod 21.]

Both middle images seem to have 7 needles. Given ***r*** = 64, there are 8 iterations of the 8 jump set Fibonacci sequence. There are 8 needles, but the inner-most iteration is quite close to the center and hard to see. As a result, 7 iterations are readily visible, and you can see that each iteration produces a smaller needle.