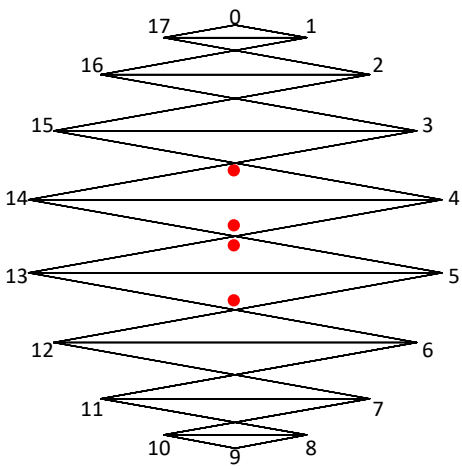
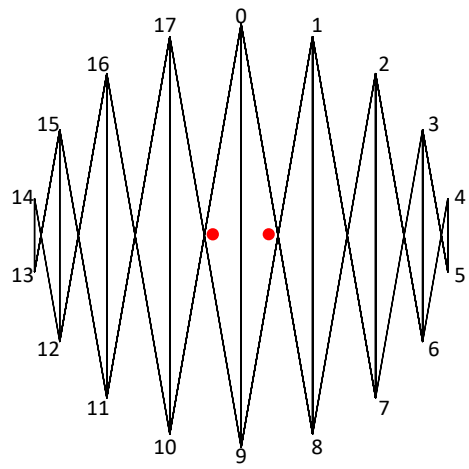
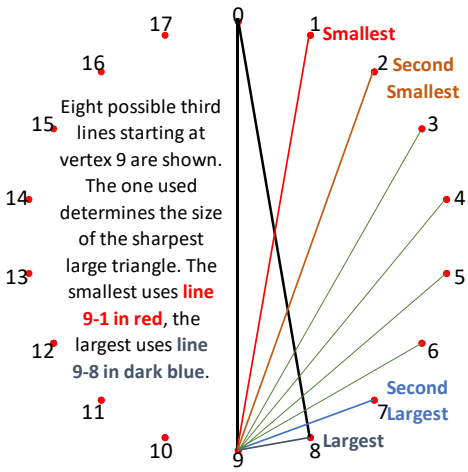
What we mean by *Largest Large Triangles* and *Smallest Large Triangles*

Whenever three distinct lines drawn from vertices of an ***n***-gon intersect at three distinct points within or at vertices of the ***n***-gon, a triangle is formed. The images created using the lines parallel from these lines creates the images which we use for counting exercises. The three lines need not create the largest triangle in an image, but at least one triple of lines does (or more than one triple does, if there are multiple largest triangles). For example, the left image is based on the smallest triangle 0-1-17, but there are four largest triangles in this image (**with red dots next to the apex angle of each**) because all are similar (have the same angles in each) and two have longest and equal bases of 4-14 and 5-13. These bases are equal in size because both span 8 vertices. Notice that all four of these largest triangles use only two of three vertices (the base vertices), the apex vertex is an interior intersection in each case. The left image uses horizontal bases. The middle image shows an alternative configuration using vertical bases. In this instance, there are two largest triangles using the diameter 0-9 as base. Since a diameter is the largest line spanning two vertices, the two largest triangles in the middle image are larger than the four in the left image. Nonetheless, given the configuration of each, all six are the largest triangles in each image. Although these are the largest triangles in both images, we will call these the ***smallest large triangles*** given ***n*** = 18. ***n*** = 18 was chosen because triangle angles are then multiples of 10° as discussed in E22.1.1.







To understand why the **red-dotted** triangles are **smallest large triangles**, consider the right image. It uses two of the lines from the middle image, the vertical diameter 0-9 and the line 0-8 used to create the sharpest angle of 180/***n***° = 10° at vertex 0. To reduce clutter, none of the other lines parallel to these two lines are shown. Instead, eight possible third lines starting at vertex 9 are shown. The intersection of each third line with the line 0-8 creates a largest triangle given this third line direction since the base in each is the diameter of the circle. The line ending at vertex 1 is clearly the smallest large triangle because it has the same base, but the height is lower at the intersection point than any of the other lines. Also, one can see by overlaid triangles that each successive line down (from vertex 9 to vertex 2, 3, …, 8) contains the prior triangle plus a new triangular part. The ***largest large triangle*** occurs when the third line spans a single vertex (from 9 to 8 in the right image which produces a triangle with angles of 10° at vertex 0, 80° at 9, and 90° at 8).

An aside on alternatives to the image at right. We could have drawn a similar version of the left image by drawing lines 0-10 and 0-9 in black then having nine possible third lines connecting vertex 10 to vertices 1 through 9. The smallest large triangle in this situation would be the triangle created by the intersection of line 10-1 and 0-9, and it would be the same size as the four in the left image. And if we had an odd ***n***, like ***n*** = 17, the right image would only need to be adjusted a bit since the line 0-9 and 0-8 would straddle the bottom of image. The largest large triangle then would be the isosceles triangle 0-8-9 which was the subject of the last chapter (there is only one largest isosceles triangle given odd ***n***).

How much Larger is the *Largest Large* than *Smallest Large* Triangle? The largest large triangle is almost twice as large as the smallest large triangle in the image at right. This is easy to see if you note that the intersection of **red line 9-1** and black line 0-8 creates an isosceles triangle. The line through this point perpendicular to 0-9 intersects 0-9 at (0,0) the midpoint of the vertical diameter. By similar triangles, a line perpendicular to 0-9 at point 9 would intersect the extension of the line 0-8 just below vertex 8 at a point twice as far from the 0-9 line and therefore twice as large and this is only marginally larger than the triangle 0-8-9. ***Note:*** Right triangle 4-5-13 is EXACTLY twice the smallest large triangle at left.