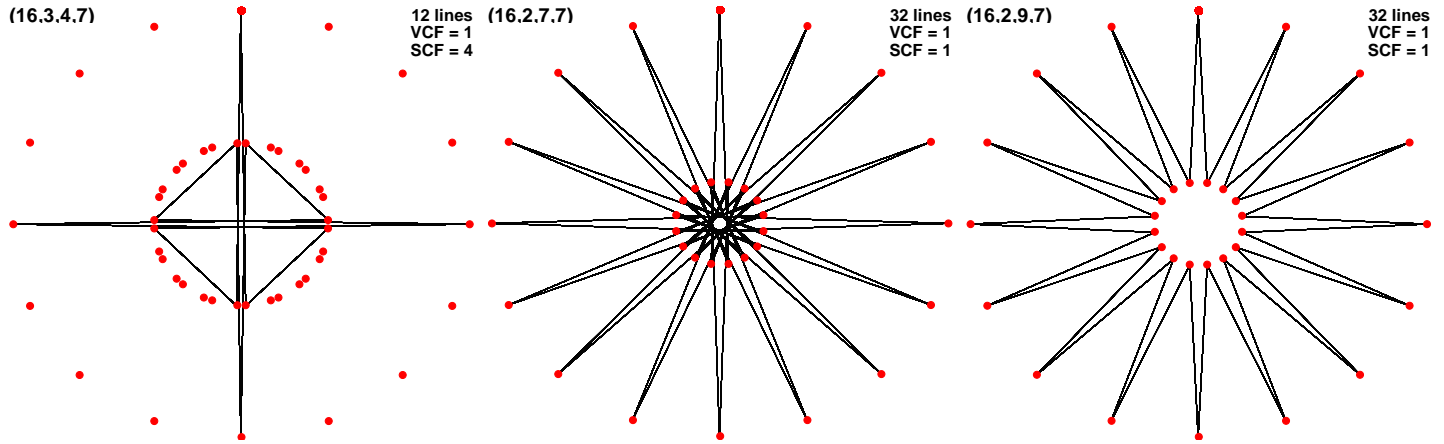
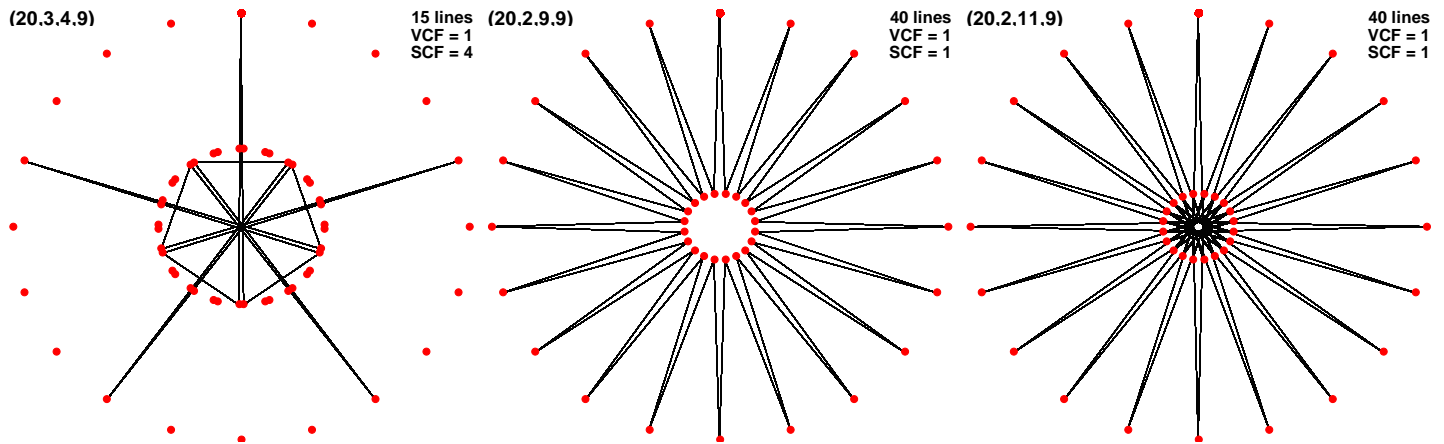


$n = 4k$ Sharpest Central Needles Stars

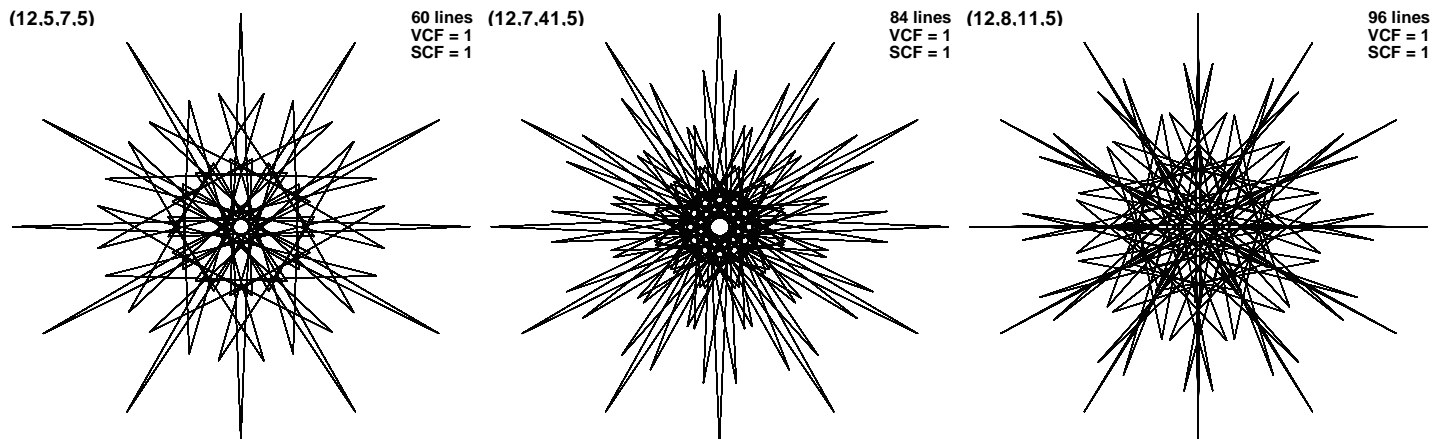
It turns out that different rules apply for even n that are divisible by 4 from those that are divisible by 2 but not 4. One reason stands out more than any other, if $n = 4k$ then $n/2 = 2k$ or an even number. This means that one vertex less is odd and if $J = 2k-1$, then $VCF = 1$. If we use the *odd sharpest central needles* rule: $n = 2k+1$, $J = k$, $S = 3$, $P = 4$, $SCF = 4$ and the resulting image uses one fourth the vertices like the $n = 16$, 4-point version to the left. Compare this to these two $n = 16$, $S = 2$, $SCF = 1$, 16-points images. The star-in-a-star ([E9.1](#)) uses the donut hole, the sunburst ([E11.5](#)) does not.



Notice the pattern here. Both have the same n , S , J and differ by P . For one, P is one less than $n/2$, for the other, P is 1 more. Interestingly, the pattern alternates. To see this most clearly, compare the next larger $n = 4k$, column by column.



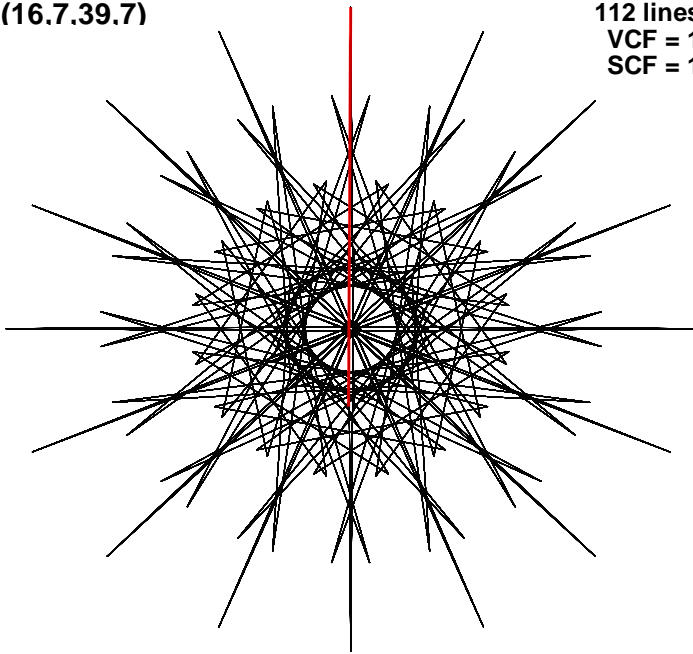
One fourth of the subdivisions are used at left, but the sunburst is now the smaller P , and the star-in-a-star is the larger P .



Each of these uses a larger S and is replicable for larger $n = 4k$ and $J = 2k-1$. The left and right images maintain S and P as shown above. The middle is a porcupine star, [E11.4](#), with $P = nS/2-1 = 2kS-1$.

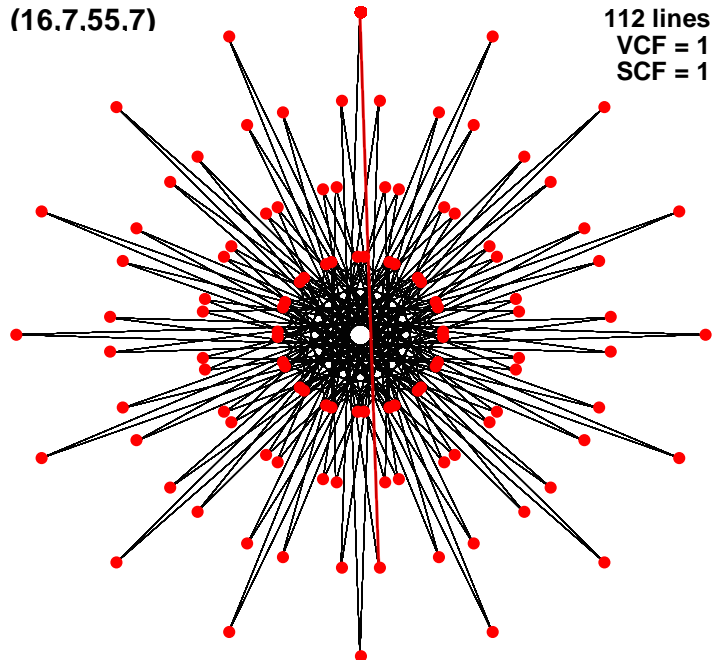
It is worth noting that not all $n = 4k, J = 2k-1$ porcupines are sharpest central needles as we see here in a comparison of two $n = 16, J = 7, S = 7$ images. Subdivision **dots** have been added to the porcupine on the right. The only way to tell is to click subdivision dots on and see whether the needle created by the first line (in **red** ending one subdivision **dot** from the bottom) and last lines contains **dots**. Both Level 3 ([E7.1](#)) pairs of **dots** are inside this needle on the right, and the left dot below the center is the end of the **first line segment for the left image shown in red**.

(16.7.39.7)



112 lines
VCF = 1
SCF = 1

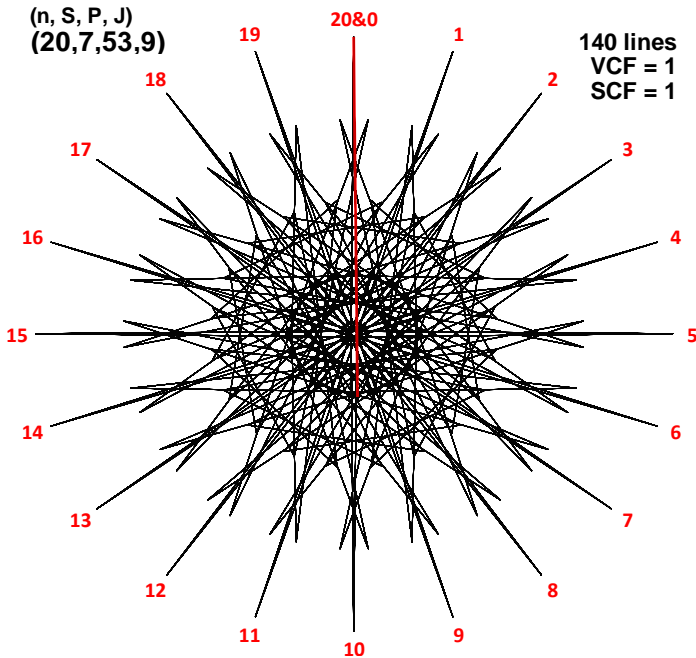
(16.7.55.7)



112 lines
VCF = 1
SCF = 1

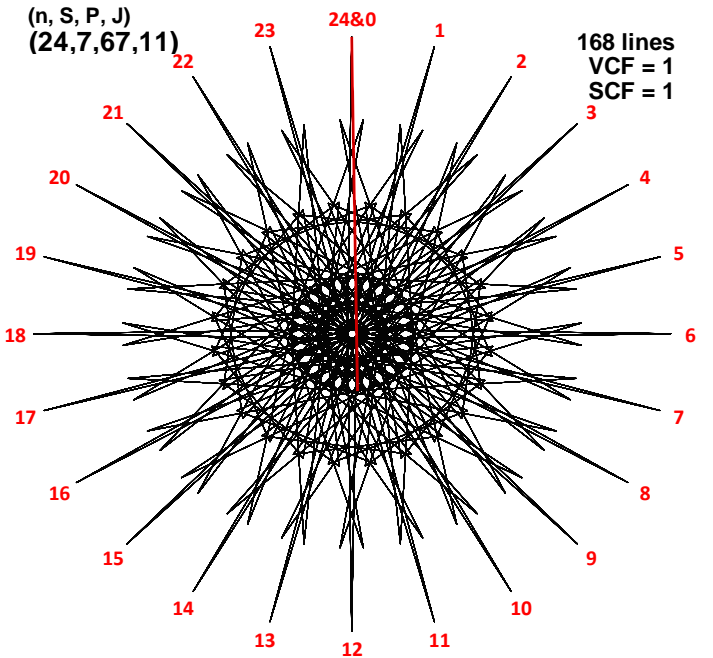
Here are the next two versions of the left image. The parameters of each model follow: $n = 4k, J = 2k-1, S = 7$. Can you figure out the pattern given P changes from 39 to 53 to 67. Predict the next value of P . [A rationale for why this is the case is provided below the images if you want to think about it prior to reading further.]

(n, S, P, J)
(20,7,53,9)



140 lines
VCF = 1
SCF = 1

(n, S, P, J)
(24,7,67,11)



168 lines
VCF = 1
SCF = 1

This works because as n increases by 4, J increases by 2, and there are 4 more lines in the VF. The end of the first line is the 4th subdivision endpoint on the VF line from vertex 3 to $n/2+2$ (which is a jump of $n/2-1$). It turns out that 4/7 of the way from vertex 3 to vertex $n/2+2$ is close to the vertical diameter. (Adding 14 is the same as adding two lines to the VF.)

It is worth noting in this analysis that the three images have first line endings on different sides of the vertical diameter. This contrasts with the [odd \$n, S = 3, P = 4\$ analysis](#) that always had the first endpoint to the right of the vertical diameter.