About *Distinguished Points*: How Many Rectangles are in a Clockface?
(How many of these rectangles are squares?)

The horizontal and vertical lines obtained by connecting vertices of a 12-gon produce the image below. There are rectangles, lots of rectangles. In fact, there are 51 rectangles of various sizes in the image. If you do not come up with that number, it is likely because you are not counting systematically. To systematically count, we use the notion of a *distinguished point*.

Definition. A *distinguished point* is something that is unique about an object.

Using Distinguished Points. It does not matter what point is chosen as long as it is unique. To show this, we use two different distinguished points to count rectangles. (Each corner of a rectangle could be a distinguished point.)

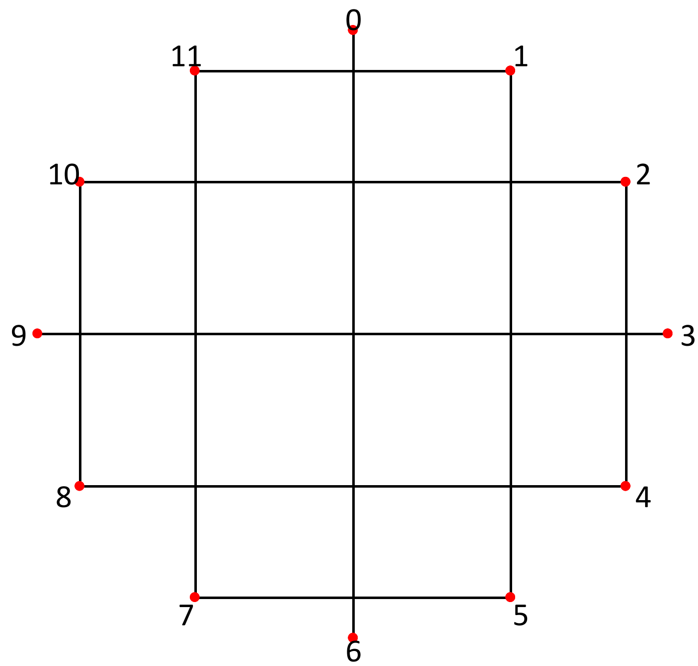
A) Use upper left corners as the distinguished point.

8, 4	counting horizontally from vertex 11
8, 8, 5, 2	counting horizontally from vertex 10
4, 5, 3, 1	counting horizontally from vertex 9
2, 1	counting horizontally from vertex 8
0	counting horizontally from vertex 7.

B) Use lower right corners as the distinguished point.

0	counting horizontally from vertex 1
2, 1	counting horizontally from vertex 2
4, 5, 3, 1	counting horizontally from vertex 3
8, 8, 5, 2	counting horizontally from vertex 4
8, 4	counting horizontally from vertex 5.

Using both methods, the sum of counts is 51.



Each method counts various-sized rectangles all at once using the distinguished feature. Here are totals by various size.

1x1 = 12; 1x2 or 2x1 = 16; 1x3 or 3x1 = 8; 1x4 or 4x1 = 4; 2x2 = 5; 2x3 or 3x2 = 4; 2x4 or 4x2 = 2.

Distinguishing Squares from Rectangles. Squares require equal length sides in addition to equal angles. If a side spans two vertices of a regular polygon it is the same length as another side spanning two vertices even if neither is a corner of the rectangle under consideration. In the image above, the length 1-11, 2-4, 5-7, and 8-10 are all the same. The rectangle created by the intersection of lines 1-5, 2-10, 4-8, and 7-11 is a 2x2 square as a result. Additionally, this square has four 1x1 rectangles, and each is a square since each side of those four 1x1 rectangles is half the length of the two vertex span. It is worth noting that half of a two vertex span is smaller than a one vertex span because the hypotenuse of a right triangle is larger than either leg.

By contrast, none of the other four 2x2 rectangles are squares because a span of four vertices (such as 2-10, for example) is less than twice the size of the span 1-11 which spans two vertices.

Using Diagonals to Find Squares. An easy way to distinguish a square from an ordinar rectangle is that at least one set of opposing vertices are part of a diameter of the circle. If both sets of opposing vertices are diameters, then the square is *centrally located*. These diameters need not end at vertices of the underlying ***n***-gon.

Five of the 51 rectangles in the image are squares. The 2x2 version is centrally located because its vertices are on the diameters 1.5-7.5 and 4.5-10.5. The four 1x1 squares are on one but not the other of these diameters.