Any Sharpest Triangles Image that is not Isosceles or Right is Obtuse

We explored sharpest isosceles triangles in the last chapter and sharpest right triangles in this chapter. Given an assumption that one of the angles in the triangle spans a single vertex, the only other possibility is that one of the remaining two angles is obtuse. The logic behind this statement is simple.

Any triangular image using vertices of a regular n-gon to create the lines can be described by the three angles thus created, E2.6.3. As shown elsewhere, these angles must be multiples of $180/n$ according to the inscribed angle theorem (or the interior angle theorem for angles not at polygonal vertices) and they must sum to n. We can consider these angles as $180a/n°$, $180b/n°$, and $180c/n°$ where a, b, and c are whole numbers which sum to n and hence can be thought of as arcs of a circle. If one angle spans a single vertex, $a = 1$. The remaining two angles must span n-1 vertices.

Odd n*.* When n is odd, $n = 2k+1$ and $a = 1$, the two remaining angles must span $2k$ vertices and $c = 2k-b$ since $b+c = 2k$. If both are equal, the result is a sharpest isosceles triangle with base angles spanning k vertices that was the centerpiece of the last chapter. If $b \neq k$, the closest we can come to isosceles without being isosceles is if one spans k-1 and the other spans k+1 vertices. The larger span creates an obtuse angle because $(k+1)/(2k+1) > ½$ so $180(k+1)/n° > 90°$. One might call this the *least obtuse scalene triangle* because all other versions will have a largest vertex span of larger than k+1.

Even n*.* When n is even, $n = 2k+2$ and $a = 1$, the two remaining angles must span $2k+1$ vertices. We just examined the closest to isosceles given this situation and that happened when b and c span k and k+1 vertices. The larger of these spans is half the circle so that the angle spanned is 90°, a sharpest right triangles image results. The closest we can come to right without being right is if one spans k-1 and the other spans k+2 vertices. The angle created by the larger of these two spans is obtuse because $(k+2)/(2k+2) > ½$ so $180(k+2)/n° > 90°$. One might call this the *least obtuse scalene triangle* because all other versions will have a largest vertex span of larger than k+2.

Put in terms from the last section, the sharpest odd isosceles triangles images and sharpest even right triangles images are both examples of *Largest Large Sharpest Triangular Images*. The largest angle in these types of images is a right angle, and that occurs when n is even. If the sharpest triangles image is not largest large, then one of the angles is obtuse.

Before examining the least obtuse scalene triangles case (second largest large), we focus on the opposite end of the spectrum, sharpest most obtuse triangles images.