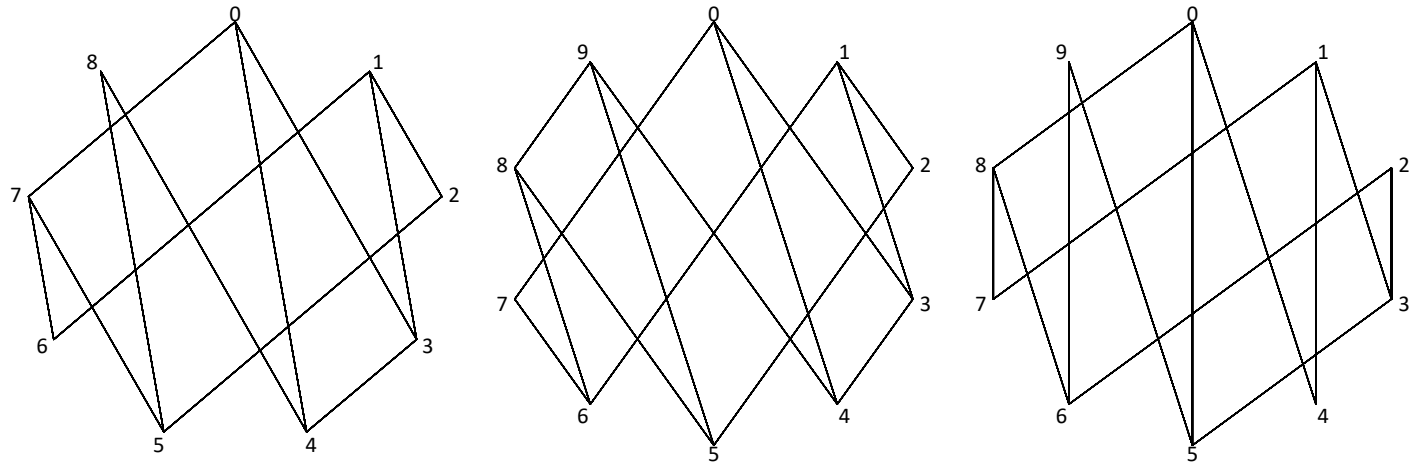
Sharpest Least Obtuse Scalene Triangles Images

We argued that the second largest large triangle creates the least obtuse scalene triangles image for both even and odd ***n***. This is easiest to see if we create an image using ***j*** = INTEGER((***n***-1)/2) which is the vertex just to the right of the bottom and ***k*** = ***j***-1, ***v*** = ***j***, and ***w*** = ***k***. The first two images are based on these parameters given ***n*** = 9 and 10 so ***j*** = 4 and ***k*** = 3 and the triangle 0-3-4 has angles 20°, 100°, 60° for ***n*** = 9 and 18°, 108°, 54° for ***n*** = 10. In this construction, the angle 0-***k***-***j*** is necessarily least obtuse having angle 180(***j***+1)/(2***j***+1)° for odd ***n*** = 2***j***+1 and 180(***j***+2)/(2***j***+2)° for even ***n*** = 2***j***+2.

The right image was created by altering two parameters for even ***n***, ***k*** = ***j***+1 and ***v*** = ***j***-1. All the triangles in the right image are similar to those in the middle since ***n*** = 10 but note now that the vertices without sharpest angles at vertices 2 and 7 are acute rather than obtuse. By contrast, the odd image at left has one of each at vertices 2 and 6. Once again we have two versions with different sized similar triangles when ***n*** is even but only one when ***n*** is odd.



Counting for odd *n* = 2*k*+1. The second largest large triangle has angles that span (1,***k***-1,***k***+1) vertices and the plateau of triangle counts is at level ***k***-1. Counts from each side increase by 1 from 0 to ***k***-2 on both sides before first encountering a vertex with a count of ***k***-1 so that 2(***k***-1) vertices have counts less than ***k***-1 and if we include one of the ***k***-1 vertices we obtain a total count of $(k-1)^2$ from these 2***k***-1 vertices according to *The Hill Formula*. The remaining ***n***-(2***k***-1) = 2 vertices are at a count of ***k***-1 each so total triangle count for odd ***n*** is T(***n***=2***k***+1) = $(k-1)^2+2(k-1) = (k-1)(k-1+2) = (k-1)(k+1) = k^2-1$ by the difference between squares formula. There are 15 triangles in the left image with plateau triangles count of 3 at vertices 0, 4, and 8. This is 1 less than the number of sharpest isosceles triangles found in the last chapter so that the left bottom image has 99 triangles (with a plateau of 3 vertices (0, 10 and 11) at 9 triangles each) while the second has 100.

Counting for even *n* = 2*k*+2. The second largest large triangle has angles that span (1,***k***-1,***k***+2) vertices and the plateau of triangle counts is at level ***k***-1. Counts from each side increase by 1 from 0 to ***k***-2 on both sides before first encountering a vertex with a count of ***k***-1 so that 2(***k***-1) vertices have counts less than ***k***-1 and if we include one of the ***k***-1 vertices we obtain a total count of $(k-1)^2$ from these 2***k***-1 vertices according to *The Hill Formula*. The remaining ***n***-(2***k***-1) = 3 vertices are at a count of ***k***-1 each so total triangle count for even ***n*** is T(***n***=2***k***+2) = $(k-1)^2+3(k-1) = (k-1)(k+2) = k^2+k-2 = k(k+1)-2$. The middle and right images have 18 triangles with plateau triangles count of 3 at vertices 0, 4, 5, and 9. This is 2 less than the number of sharpest right triangles found earlier in this chapter so that the third bottom image has 108 triangles (with a plateau of 4 vertices (0, 10, 11 and 21) at 9 triangles each) while the right image has 110.