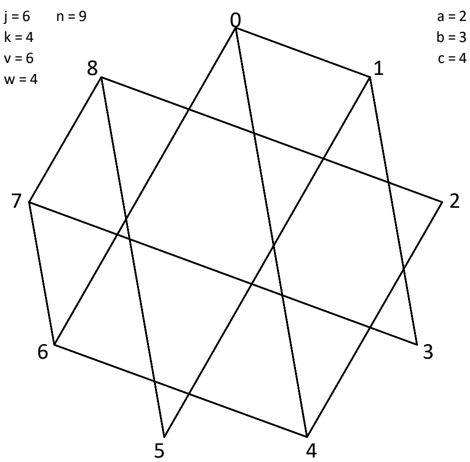
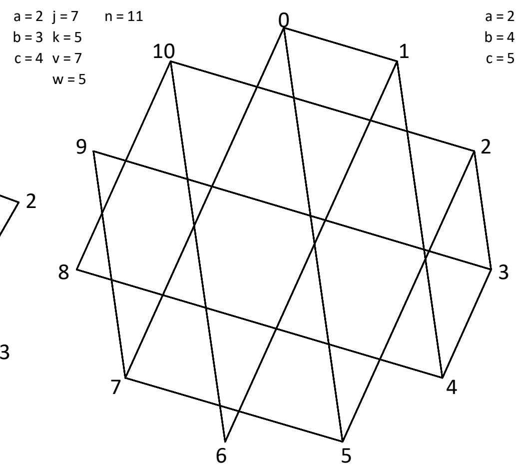
## A *Worked Example* Using *Excel* to Analyze Images: *Second Sharpest Odd Acute Scalene Triangles*

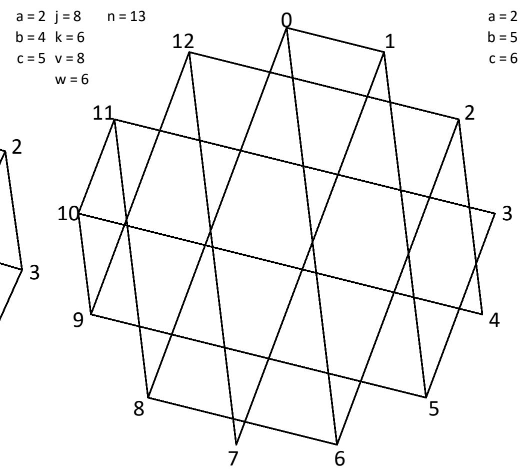
Once you determine a rule for the type of image sequence you wish to examine, you can use the ***j***, ***k***, ***v***, and ***w*** cells in the **General Triangles** *Excel* file to express the rule algebraically. Then as you vary ***n***, you can see the images change in response to the rule. The rule can either have ***j***, ***k***, ***v***, and ***w*** be functionally related to ***n*** or not.

Thus far we have analyzed sharpest triangles because, by construction, the sharpest vertex of any resulting triangle must be a vertex of the ***n***-gon. The sharpest vertex is easy to distinguish from other vertices of a triangle and is therefore used as the *distinguished point*. Given this, all we needed to do is to count triangles using vertices of the ***n***-gon. Once we relax that assumption, we must consider interior sharpest vertices. We start with the simplest modification possible based on odd ***n*****. Suppose you want the smallest angle to span 2 vertices, and the image to be acute and scalene.** The smallest ***n*** you would want to consider here is 9 because 2+3+4 = 9. The largest span (***k*** = 4) is less than ***n***/2 so the triangle is acute.

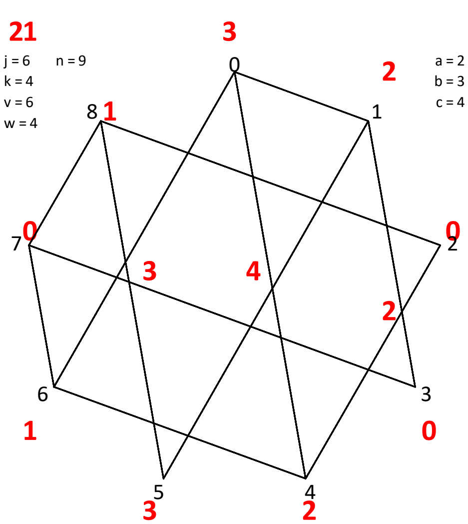
**Functionally Related Parameters.** Since we are interested in odd ***n***, it seems natural to write ***n*** = 2***k***+1 and let ***k*** from the *Excel* file do "double duty" here, so we set ***k*** = INT(***n***/2). When ***n*** = 9, ***k*** = 4 using this function and it creates a line from the top vertex to the bottom right vertex. Next, set ***j*** = ***k***+2, ***v*** = ***j***, and ***w*** = ***k***. The triangle 0-***k***-***j*** has sharpest angle at 0 (of 360/***n***°) and largest angle at ***j*** (of 180***k***/***n***°). The first three odd images created using these functional relationships are shown below. (It is worth noting that this also produces *second sharpest scalene right triangles* for even ***n*** ≥ 10.)

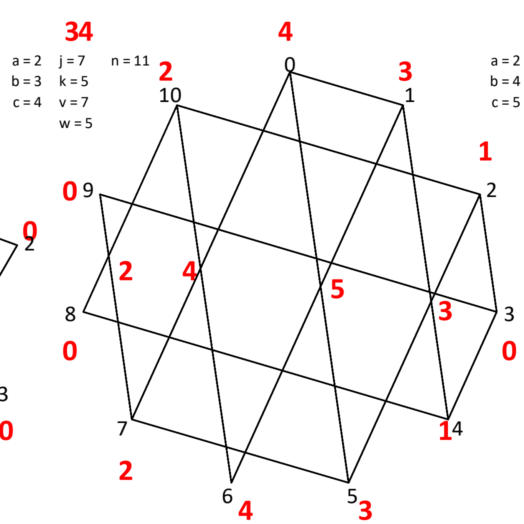


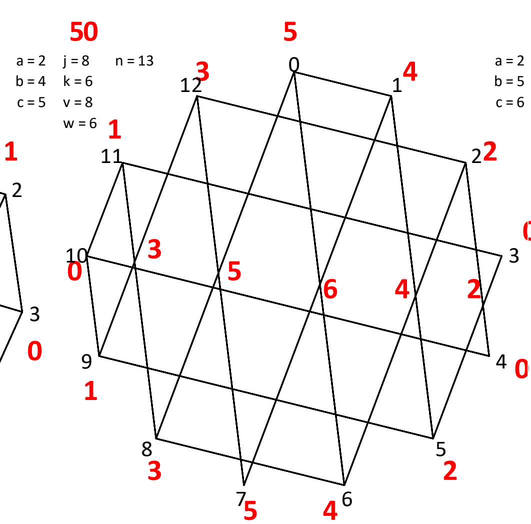




**Transferring Images for Counting.** The *Excel* file has instructions for transferring an image out of *Excel* into *Word*. This is a useful intermediate step even if you want to use the image back in *Excel* because you can transfer the image onto an *Excel* sheet that is set up as a piece of graph paper. This allows you add **apex counts** near sharpest angle locations on both the interior and perimeter of the image. The images below show the result of this process (which will be discussed next). Counts are noted in **red** with the sum of counts in **red** at upper left for each image. You need not be that careful about exact placement if the intersection point is reasonably unambiguous. You are looking for patterns in the numbers.



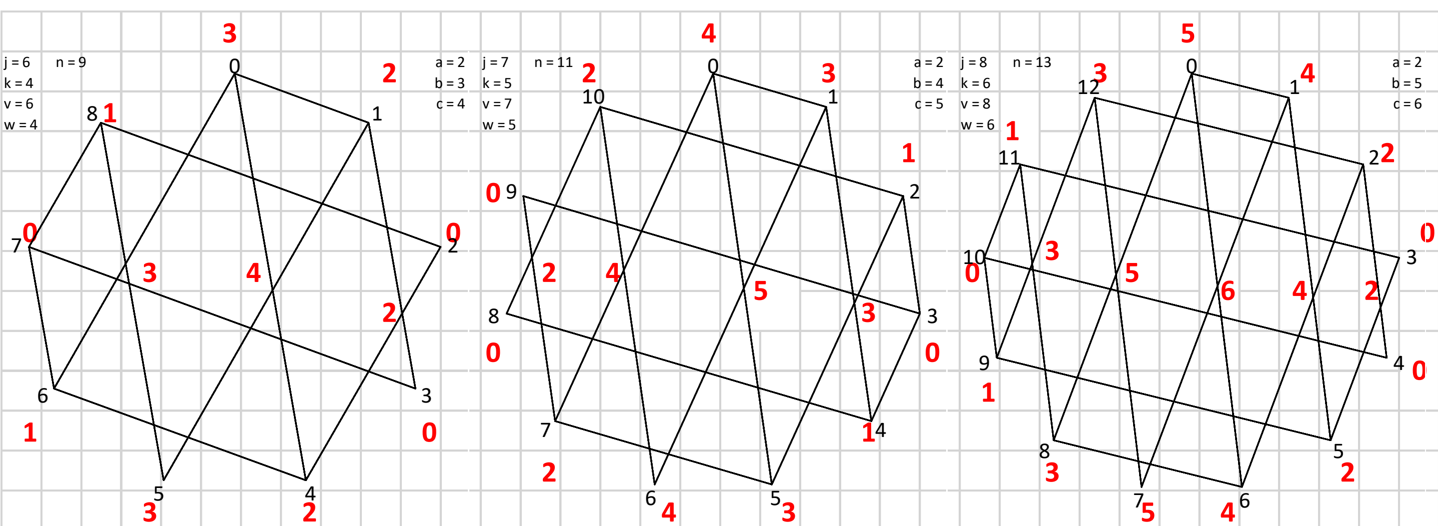
**Creating Graph Paper.** When you open a fresh sheet in *Excel*, rows are set to 20 pixels, columns are set to 64 pixels, with a default font of Calibri 11. Create graph paper by following these instructions.

1. Click the gray triangle, ◢, above row **1** and to the left of column **A** labels to highlight the entire worksheet.
2. Move your mouse to the **A|B** border until you see the dark black vertical line **|** with horizontal arrows ↔ .
3. Hold the mouse down and reduce column width to 20 pixels.
4. Click the gray triangle to highlight the entire worksheet.
5. Change font to bold (click **B** in the font area) and change font color to **red**.

The **red font** allows you to distinguish between **apex counts** and vertex labels (which are part of the image (and can be omitted) but are often useful to have even though they make the image a bit busy). If you want numbers larger than **99** in a cell, it makes sense to change cell formatting to *shrink to fit*. To do this, click the bottom right of the *Font* area of the **Home** ribbon to bring up the **Format Cells** pop-up menu then choose the **Alignment** tab and click **Shrink to Fit**.

***Excel* to *Word* to *Excel*.** Once an image has been transferred into *Word* using **Paste Special**, **Picture Enhanced Metafile** as noted in the **General Triangles** *Excel* file, it can be resized. Resize in *Word* using the **Picture Format** ribbon which is visible once the image is clicked on in *Word*. (These three images were put side by side by setting *Width* to 2.45".) Once the image is an appropriate size, click on the image in *Word* then click **Copy** and click on the graph paper sheet in *Excel*. Click on a cell you want to have as the upper left corner and click **Paste**. This was done three times to transfer the three images to the *Excel* graph paper by clicking on cells **B3**, **N3**, and **Z3**. The image below shows cells **B2:AK14**.

**Apex Count Locations.** The images are hollow, and they are floating above the graph paper in *Excel*. Apex cell counts are typed into cells near interior and perimeter apexes. This is done for each image using the *Arrow* keys on your keyboard. Type the apex count for each apex. You can use the alignment keys to move numbers to the left or center of a cell (right is default for numbers). For example, the **4** in cell **H8** in the middle of the left image is more visible if it is left justified. Sometimes it is useful to merge cells like the **3**, **6**, **4**, and **2** in the interior of the right image, or **5** in the middle image.




**Summing Apex Counts.** One of *Excel's* automatic functions is that it will provide information (including sum) about a group of cells that are highlighted. For example, if you start in cell **B2**, hold the *Shift* key down then arrow over to column **M** and down to row **14** you will see the sum noted at the bottom right corner of the *Excel* sheet as *Sum*: **21**. Similarly, the sum from **N2:Y14** is **34** and the sum from **Z2:AK14** is **50**. These numbers were added to upper left of each image, and **Gridlines** is clicked off from the **View** ribbon. The result is the set of three images on the previous page.

**Total Triangles for *Second Sharpest Odd Acute Scalene Triangles* Images.** Looking at the images we see that there are the numbers from 1 to $k$-1 at top and bottom along the perimeter and the numbers from 2 to $k$ across the center. Using the [Triangular Numbers] formula, this leads to the total number of *second sharpest odd acute scalene triangles*, **T**:

$$\mathbf{T}(n = 2k+1) = 2\Delta_{k-1} + \Delta_k - 1 = 2(k-1)k/2 + k(k+1)/2 - 1 = k(2(k-1) + (k+1))/2 - 1 = k(3k-1)/2 - 1 = (3k+2)(k-1)/2.$$