Alternative Ways to Portray Numbers

At one level, a number is simply a number. The number 13, for example, represents 13 things … vertices, dots, squares, etc. But it can also be viewed in different ways, and the way that you portray that number may help you to understand an image sequence more deeply. (It is worth mentioning that there is nothing unique about 12 or 13, we could have examined these same kinds of issues using other numbers.)

Searching for Similarity. Geometric and numeric patterns often repeat themselves as ***n*** changes. The images or numeric patterns are not the same, but you can see that they are related. For example, if you are doing a counting problem and you have apex counts where some of the cells are 0 (because there are triangle bases but not apexes at that vertex) these might look similar every second, or fourth value of ***n***. This would lead you to count by 2s or 4s. The numbers 2 and 4 are not the only possible repeating values but they are the most common for the problems we typically see in **PwP**. We will encounter situations in **PwP** where counting by 6s and counting by 8s also makes the most sense.

Counting by 1s. There is no need to dwell on this as it is how we were introduced to numbers. Sometimes we will see an image that changes smoothly as an individual parameter such as ***n***, changes by 1.

Counting by 2s. Sometimes the pattern repeats every 2 numbers. This is the basis of evens and odds.

12 is an even number. But if we consider a sequence of even numbers, *Which even number is it?*

Here it makes sense to portray ***n*** as ***n*** = 2***k*** since ***k*** = 1 produces 2, the first even number (greater than 0), ***k*** = 2 produces 4, the second and so on. Setting 12 = 2***k*** and solving for ***k*** we see that 12 is the 6^{th} even number.

On the other hand, a 12-gon is one in a sequence of even polygons. *Which even polygon is it?*

Here it makes sense to portray ***n*** as ***n*** = 2***k***+2 since ***k*** = 1 produces 4, and a square is the first even polygon, ***k*** = 2 produces a hexagon, and so on. Setting 12 = 2***k***+2 and solving for ***k*** we see that 12 is the 5^{th} even polygon.

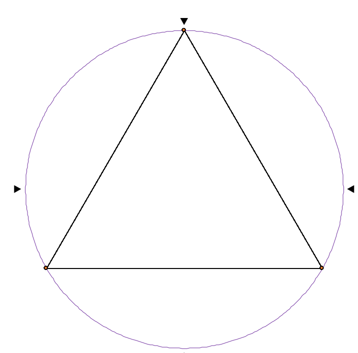
13 is an odd number. But if we consider a sequence of odd numbers, *Which odd number is it?*

Here it makes sense to portray ***n*** as ***n*** = 2***k***-1 since ***k*** = 1 produces 1, the first odd number, ***k*** = 2 produces 3, the second and so on. Setting 13 = 2***k***-1 and solving for ***k*** we see that 13 is the 7^{th} odd number.

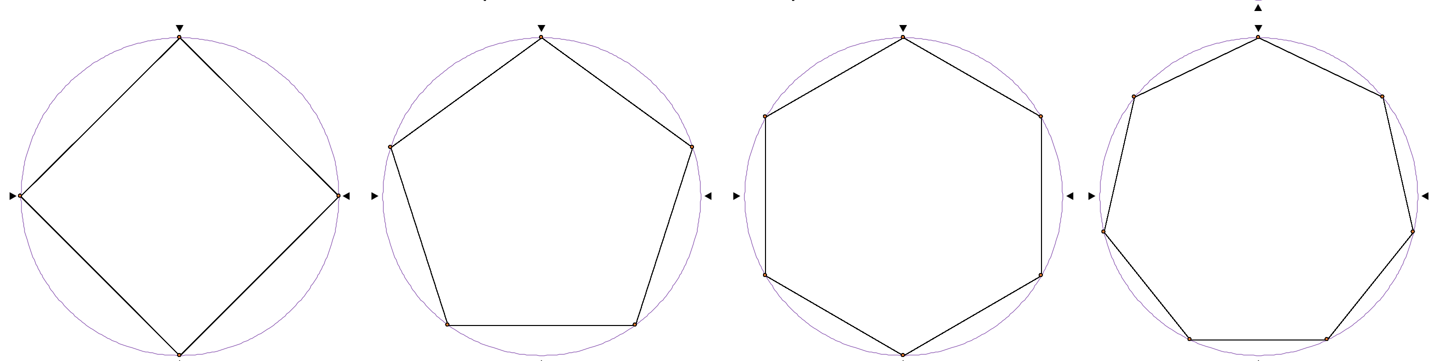
On the other hand, a 13-gon is one in a sequence of odd polygons. *Which odd polygon is it?*

Here it makes sense to portray ***n*** as ***n*** = 2***k***+1 since ***k*** = 1 produces 3, the first odd polygon is a triangle, ***k*** = 2 produces a pentagon, and so on. Setting 13 = 2***k***+1 and solving for ***k*** we see that 13 is the 6^{th} odd polygon.

Counting by 4s. A simple example of counting by 4s based on a geometric pattern is to consider what happens with an ***n***-gon's sides at the 3 o'clock and 9 o'clock positions shown as ◂ and ▸ on the circle (at ¼ or the right-most and ¾ or the left-most position moving clockwise from the top). There are four possibilities before the pattern repeats itself as shown in the similarity of the 3-gon and 7-gon images to the right.



If ***n*** = 4***k***, like 4, 8, …, there are points since ***k*** and 3***k*** are vertices at these locations.
If ***n*** = 4***k***+1, like 5, 9, …, the line is sloped upward at ¼ and downward at ¾ around.
If ***n*** = 4***k***+2, like 6, 10, …, then the polygonal lines at these points are both vertical.
If ***n*** = 4***k***+3, like 3, 7, …, the line is sloped downward at ¼ and upward at ¾ around.



Starting Values. It is worth noting that another common way to describe these series is to consider the starting value. We mentioned this in counting by twos, but it is worth building this out a bit more fully. For example, suppose the first image you want to discuss is an octagon, $n = 8$, but you want to examine every fourth n after that. Then, it might be better to say $n = 4k+4$ so that $k = 1$ is the octagon. Similarly, if you want to talk about polygons 3, 7, 11, … then it may make more sense to say $n = 4k-1$ rather than $4k+3$ because then the triangle is represented as $k = 1$ rather than $k = 0$.

A Nod to Modular Arithmetic. If you have not encountered modular arithmetic before, what we have discussed above can be put in modular terms. It is not central to the analysis of counting issues, but it is worthwhile to at least know that this area of mathematics exists. Modular arithmetic focuses attention on remainders, the 0, 1, 2, and 3 encountered when we were categorizing the figures above, mod 4. Chapter 23 of ESA discusses modular arithmetic basics; this is the first section of that chapter. It is also of seminal importance to the string art materials encountered in PART II of **PwP**, although it is perfectly fine to enjoy many of the images discussed there, as well as images you create yourself, without knowing about modular arithmetic.