A Compendium of Number Patterns

These number patterns are often seen in counting exercises on polygonal vertices and are discussed at various points in **PART I.** They are provided here without proof. Geometric interpretations of many of these equations are provided elsewhere within **PwP**.

One-Sided Patterns	Example	Sum	General	Sum	Description
All numbers	1, 2, 3, 4, 5	15	1, 2, …, k	$k(k+1)/2$	Triangular Number, Δ_k
Even numbers	2, 4, 6, 8, 10	30	2, 4, …, $2k$	$k(k+1)$	$2\Delta_k$
Odd numbers	1, 3, 5, 7, 9	25	1, 3, …, $2k-1$	k^2	Square by Gnomons
Side-to-Side Patterns					
Hill by 1s	1, 2, 3, 4, 3, 2, 1	16	1, 2, …, k, …, 2, 1	k^2	The Hill Formula, H_k
Hill by 2s	2, 4, 6, 8, 6, 4, 2	32	2, 4, …, $2k$, …, 4, 2	$2k^2$	$2H_k$
Odd Hill	1, 3, 5, 7, 5, 3, 1	25	1, 3, …, $2k-1$, $2k-3$, …, 1	$k^2 + (k-1)^2 = 2k(k-1)+1$	2 Squares by Gnomons
Even/**Odd**	2, 4, 6, **7**, 5, 3, 1	28	2, 4, …, $2k-2$, $\mathbf{2k-1}$, …, 1	$k(2k-1)$	Odd Δ Number, Δ_{2k-1}
Even/Odd	2, 4, 6, **8**, 7, 5, 3, 1	36	2, 4, …, $\mathbf{2k}$, $2k-1$, …, 1	$k(2k+1)$	Even Δ Number, Δ_{2k}
Modified Hills					
2-flat top	1, 2, 3, 4, 4, 3, 2, 1	20	1, 2, …, k, k, …, 2, 1	$k(k+1)$	$2\Delta_k$ or $2H_k+k$
3-flat top	1, 2, 3, 4, 4, 4, 3, 2, 1	24	1, 2, …, k, k, k, …, 2, 1	$k(k+2)$	
j-flat top			1, 2, …, k, …, k, …, 2, 1	$k(k+j-1)$	

Internal Arc Adjustments. Here are some common adjustments seen in internal arc counts.

A. Internal apex counts start at 2 (or more) on a side because internal apexes have at least one triangle on each side. If the rest of the side-to-side pattern is clear, simply subtract the start-up values from the sum created by the formula.

B. Concurrent points reduce the triangle count for that apex since one less base is used if that base passes through the lines that create the apex in the first place. Tests for concurrence are discussed elsewhere.

C. Sometimes an internal pattern needs to be adjusted in the middle. Here are two examples (which are purposely small so you can see the result both ways).

1. If the pattern is 3, 5, 6, 5, 3 it is easier to consider this $3^2+4^2-3 = 22$ by adding 1 to 6 to get the next odd number and 1 at each end (because of **A** above) then apply Square by Gnomons twice and subtract 3 from the result.
2. If the pattern is 2, 4, 6, 6, 6, 4, 2 it is easier to consider this $2\cdot 4^2-2 = 30$ by adding 2 to 6 to get an 8 in the middle, then factor out the 2 and use the Hill Formula and subtract 2 from the result.

D) If the smallest angle spans more than 2 vertices there will be multiple internal arcs of apex counts. You may want to consider whether a counting pattern exists across pairs of arcs instead of along a single arc.