An Introduction to Counting Strategies using Parallel Lines

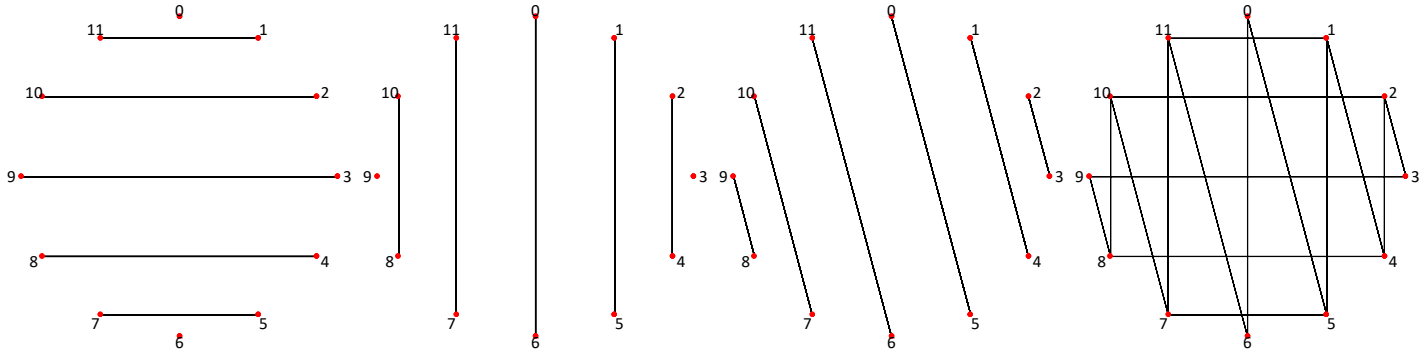
The Parallel Lines portion of Playing with Polygons is centered on the interplay between geometric and numeric patterns that occur using the vertices of regular polygons as endpoints of line segments. A variety of images can be obtained using these vertices, even if we do not subdivide the line segments connecting vertices like we do with string art images.

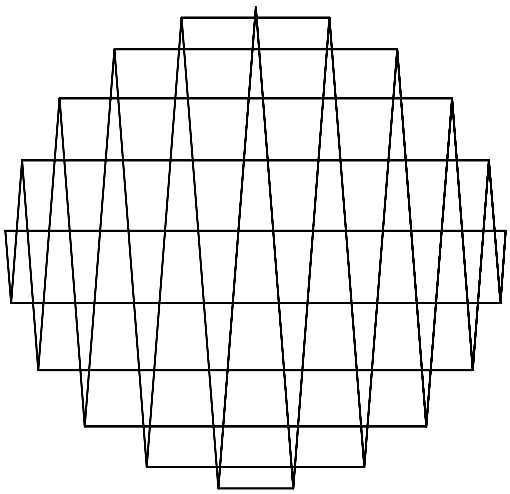
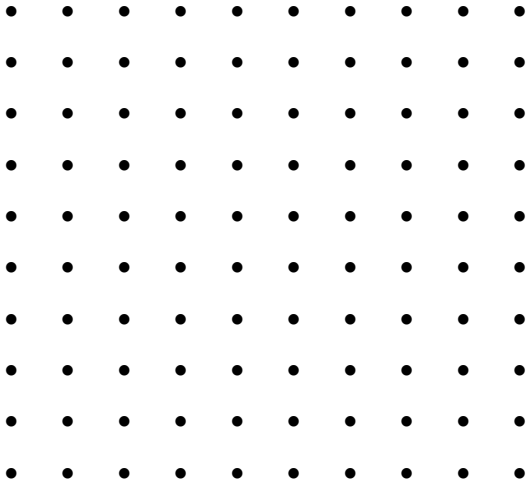
Parallel Lines. Connect any two vertices of a regular polygon and you obtain a line pointed in a specific direction. If you move one vertex over on the same side of that line, you will obtain a parallel line if there is more than one vertex on that side. This sounds abstract but it is easy to see if you think of a clockface (***n*** = 12) in much the same fashion that we did in the **Four Color Clock Arithmetic** chapter of **ESA**. The easiest lines to see are horizontal and vertical, but this works for lines other than these as well. We start by looking at horizontal and vertical lines.

Horizontal Lines. If you look at a clock, you will readily see that 11 and 1 are both at the same height. The same goes for other pairs of numbers on opposite sides of the vertical line between 0 (12 if you look at a clockface) and 6. Suppose the first line you drew was between 10 and 2. There are 3 vertices above this line so you can draw a line from 11 (10+1) to 1 (2-1) and it is also flat. You could not follow this pattern again because there is only one vertex between 11 and 1, namely 0, or 12 o-clock. But you can also draw lines below 10 to 2, in fact, as you can see at left, you can draw three such lines (9-3, 8-4, and 7-5 where the - signifies line segment).

Vertical Lines. The same argument goes for vertical lines except here we think in terms of left and right rather than above and below. The second figure results.

Other Parallel Lines. Suppose you start with the line 0-5 and then move to the left and down and the right and up. The third figure results. In this instance, all vertices are used, unlike at first and second. But had the initial line been 0-4, vertices 2 and 8 would not have lines attached, just like 0 and 6 did not in the left image or 3 and 9 in the middle image.



The right image simply puts together the first three images. Triangles result, lots of triangles. This portion of the book examines systematic methods for counting these triangles. It does so by creating links to other methods of counting.

Counting Questions. Counting exercises ask questions like: ***How many triangles of various sizes and orientations do you see in the image on the left?*** It turns out that the answer to that question is the same as the answer to the much easier question: ***How many dots do you see in the image on the right?*** To explore that link, read on.