An Overview of Diameters of an *n*-gon

Since a regular ***n***-gon is based on ***n*** vertices equally placed around the circumference of a circle, it is not surprising that we can talk about diameters of ***n***-gons just like we talk about diameters of a circle.

A diameter of a circle is any line from side to side that passes through the center of the circle. By construction, all diameters pass through the center of the circle.

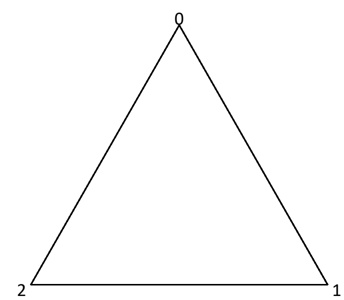
Definition. *A diameter of an **n**-gon* is a line that passes through the center of the circle and a vertex of the ***n***-gon.

To be a diameter of an ***n***-gon, it must be the case that there are the same number of vertices between the endpoints on both sides. Given ***n*** vertices total, there must be ***n***/2 vertices on either side of these endpoints.

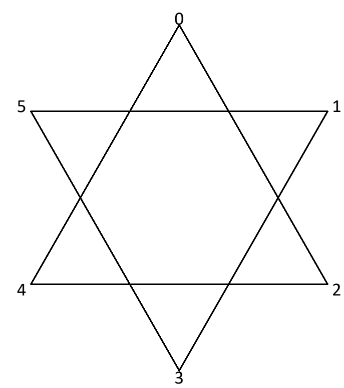
How many *n*-gon diameters are there? That depends on what you mean. If you require that both endpoints of the diameter are vertices of the ***n***-gon then the answer is 0 or ***n***/2. If you only require one endpoint to be an ***n***-gon vertex, then the answer is ***n*** or ***n***/2. Both interpretations are useful in analyzing images in **PwP**.

*Even **n**.* When ***n*** is even then ***n*** = 2***k***, and there are ***n***/2 = ***k*** diameters of an ***n***-gon since every vertex ***j*** is matched with another vertex ***k*** vertices away from ***j***, and that vertex is the other end of the j^{th} diameter. If ***j*** < ***k***, then ***j***+***k*** < ***n*** and ***j***+***k*** is also a vertex of the ***n***-gon; if ***j*** ≥ ***k***, then ***j***-***k*** ≥ 0 and ***j***-***k*** is also a vertex of the ***n***-gon.

An example. When ***n*** = 12, there are 6 diameters using vertices of the 12-gon, a vertical diameter starting at 0 and a horizontal diameter starting at **3** together with upward sloping diameters starting at **1** and 2 and downward sloping diameters starting at 4 and **5** (the bolded 12-gon diameters **1**, **3**, and **5** are seen in the third image). This may be the most natural way to conceptualize these diameters because we typically examine the ***n***-gon starting at the top and continue our examination clockwise around the vertices. But sometimes those same six diameters can be visualized by starting at vertices 6 and going to **11** (or going from **11** to 10, **9**, 8, **7**, and 6). There are ***n***/2 diameters either way.

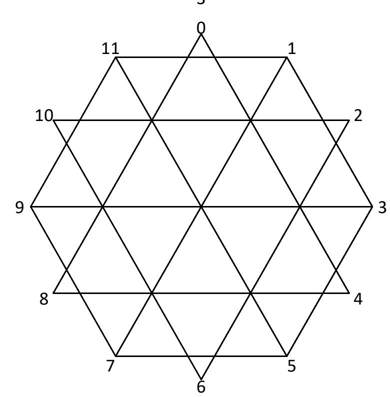


*Odd **n***. If we require the diameter line to be from vertex to vertex, then there are no diameters when ***n*** is odd. An odd ***n*** means we can write it as ***n*** = 2***k***+1, and the closest we have to equal numbers of vertices on each side of a line between two vertices is ***k*** on one side and ***k***+1 on the other.



On the other hand, if we allow the "other" end of a diameter starting at vertex ***j*** to be a non-vertex, then there are ***n*** diameters passing through the ***n*** vertices 0 to ***n***-1 and the midpoint between vertices ***j***+***k*** and ***j***+***k***+1 if ***j*** < ***n***/2 and ***j***-***k*** and ***j***-***k***-1 if ***j*** > ***n***/2. So, for example, the three triangular diameters are 0-1.5, 1-2.5, and 2-0.5.

Non-Vertex Diameters. At times it will make sense to talk about diameters of the ***n***-gon in which neither endpoint is a vertex of the ***n***-gon. This is most commonly the case when discussing lines of symmetry or internal intersection points that are on a diameter of the circle containing the vertices of the ***n***-gon. For example, a 6,2-star has three lines of symmetry as well as interior intersection points on lines 0.5-3.5, 1.5-4.5, and 2.5-5.5, and three lines of symmetry on vertex diameters 0-3, 1-4, and 2-5.



Diameters and Concurrence. If three distinct diameters intersect, they do so at the center. This is a point of concurrence. But as this 12-gon equilateral triangles image shows, it may not be the only point of concurrence. There are 7 points of concurrence, six at the corners of the internal hexagon and one in the center. Notice that here, all internal intersections on diameters 1-7, 3-9, and 5-11 are points of concurrence.

From the perspective of counting strategies in **Part I** of **PwP**, concurrent points may disrupt counting patterns on interior arcs. Consider up or down corners to be apexes (distinguished points) with horizonal bases. There are 3 arcs, one above and one below 3-9 which have apex counts 2, 3, 3, 2, and 3-9 which has apex counts of 2, 4, 2.