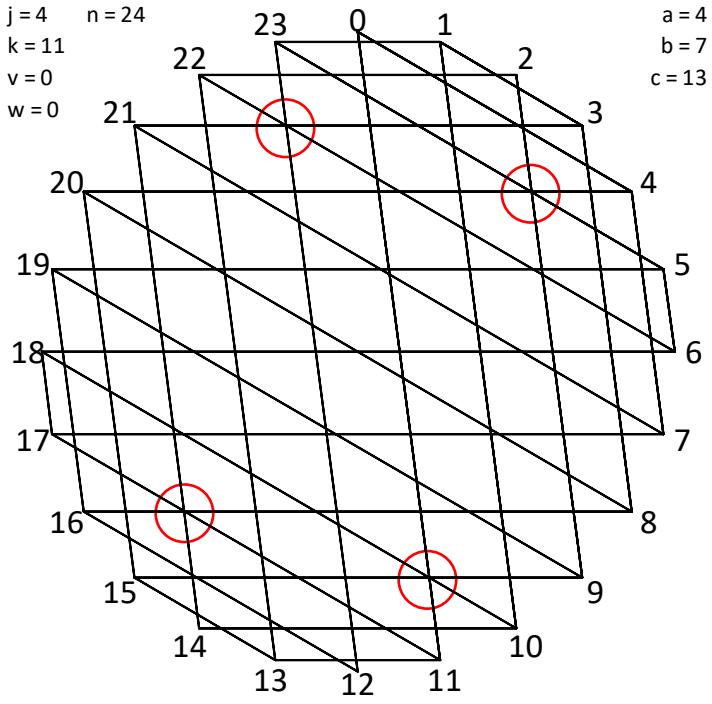
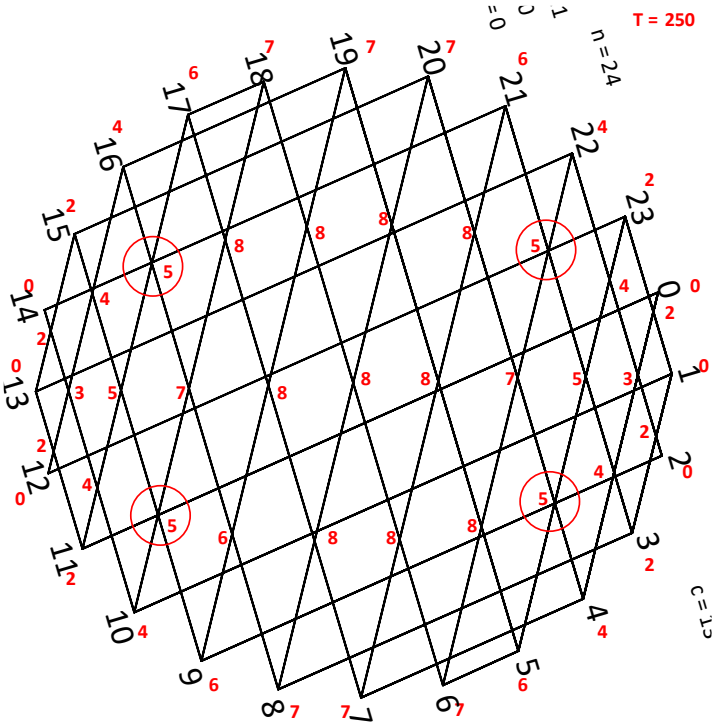
Concurrent Points can Disrupt Interior Apex Count Patterns

The images show four off-diameter apparent interior points of concurrence circled in red, and the table shows that these are concurrent points. Due to 180° rotational symmetry, only the top 2 need to be checked. The right image rotates the left to distinguish apexes more easily (easy to do in *Excel*) and marks up apex counts to find that there are **250** triangles in the image. Because the smallest angle spans 4 vertices there are 3 interior arcs of apex counts (although the middle arc is on the diameter spanning vertices 1-13 which is not part of the image). And because the middle angle spans 7 vertices, vertex apex counts top out at 7 and interior apex counts top out at 8, just as we saw in the 0-5-7 discussion.

Of interest here is the pattern of apex counts on the upper and lower arcs which is interrupted by **5** rather than **6** at points of concurrence. Without these **5**s we would have an even progression then a flat top at **8** viewed from both sides.





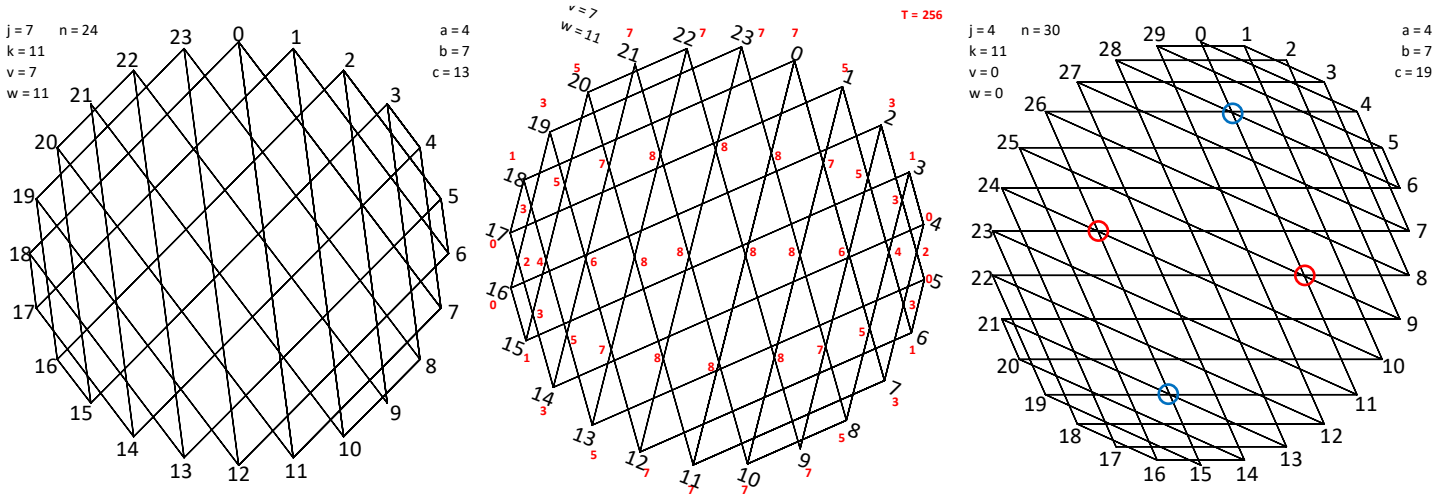
This examines four off-diameter concurrent points on an ***n*** = 24 image based on ***JKVW*** = (4,11,0,0).

4 j_0			j_0-(***n***-j_0) is a horizontal line in the ***n***-gon, vertical here.	**3 j_0**			j_0-(***n***-j_0) is a horizontal line in the ***n***-gon, vertical here.
24 ***n***				24 ***n***			
π fraction	x	y	Solve for x in	π fraction	x	y	Solve for x in
0.3333333	**0.5**	0.866025	mx+b = cx+d	0.2500000	**0.70711**	0.707107	mx+b = cx+d
			x* = (d-b)/(m-c)				x* = (d-b)/(m-c)
This IS a concurrent point.			**0.5** x*	**This IS a concurrent point.**			**0.70711** x*
2 *j*			Check: Same y at x*.	**23 *j***			Check: y = 0 at x*.
π fraction	x	y	0.548188 mx+b	π fraction	x	y	-0.22474 mx+b
0.1666667	0.866025	0.5	0.548188 cx+d	1.9166667	0.965926	-0.25882	-0.22474 cx+d
9 *j*			**Line y = mx+b from 0-8**	**12 *j***			**Line y = mx+b from 3-11**
π fraction	x	y	Δy/Δx=m -0.13165	π fraction	x	y	Δy/Δx=m -0.1317
0.7500000	-0.70711	0.707107	y_0-mx_0=b 0.614014	1.0000000	-1	0.000000	y_0-mx_0=b -0.1317
5 *j*			**Line y = cx+d from 2-14**	**6 *j***			**Line y = cx+d from 10-18**
π fraction	x	y	Δy/Δx=c -1.73205	π fraction	x	y	Δy/Δx=c -1.7321
0.4166667	0.258819	0.965926	y_0-cx_0=d 1.414214	0.5000000	6.13E-17	1	y_0-cx_0=d 1
23 *j*			With 180° rotational	**22 *j***			3 and 4 on 24 are
π fraction	x	y	symmetry, each	π fraction	x	y	octagon and
1.9166667	0.965926	-0.25882	side checks 2 pts.	1.8333333	0.866025	-0.5	hexagon vertices

In trying to generalize this image (perhaps as a class of image with spanned vertices of 4, 7, and ***n***-11 where ***n*** > 18 so that the largest angle exceeds 7 with horizontal base), we should simply consider the concurrence points to be special cases. If we ignore these four points, the total apex counts based on a formula for this image type would provide by 4 too many triangles for ***n*** = 24 shown above, one for each of the concurrence points but the formula would be readily derived based on patterns seen elsewhere. (As an aside, note that there are four two-line vertices in this image at 0, 2, 12, and 14. This distinguishes this image from the alternative 4, 7, 13 image discussed next.)

As noted elsewhere, there are typically two versions of even ***n*** images with the same spanned vertices. In contrast with the one we have been examining, this one has only two two-line vertices, 9 and 21 because both ***j*** and ***k*** are odd. The left and middle image below show similar triangles (since the angles still span 4, 7 and 13 vertices). As expected, the vertex apex counts and interior apex counts have different patterns in this instance and the totals are close to one another (**256** versus **250**), but the point to notice is that the interior apex counts now have a more easily recognizable pattern because there are no concurrent points that disrupt the pattern.

Visual inspection together with *Excel* checks for apparent concurrence points for ***n*** = 19 to 30 found only one additional 4-7-(***n***-11) image with concurrent points. There are a pair of them on ***n*** = 30 as well as two close misses as we see in the image to the right and the table. The concurrent points are circled in **red**; the near misses are circled in **blue**.



This examines both apparent concurrent points on an ***n*** = 30 image.

4 ***j*** $_0$			***j*** $_0$-(***n***-***j*** $_0$) is a horizontal line in		**8** ***j*** $_0$			***j*** $_0$-(***n***-***j*** $_0$) is a horizontal line
30 ***n***			the ***n***-gon, vertical here.		**30** ***n***			in the ***n***-gon, vertical here.
π fraction	x	y	Solve for x in		π fraction	x	y	Solve for x in
0.2666667	**0.669131**	0.743145	mx+b = cx+d		0.5333333	**-0.10453**	0.994522	mx+b = cx+d
			x* = (d-b)/(m-c)					x* = (d-b)/(m-c)
This is not a concurrent point			**0.666307** x*		**This is a concurrent point.**			**-0.10453** x*
0 ***j***			Check: Same y at x*.		**0** ***j***			Check: Same y at x*.
π fraction	x	y	**0.14857** mx+b		π fraction	x	y	**0.491768** mx+b
0	1	0	**0.14857** cx+d		0.0000000	1	0	**0.491768** cx+d
11 ***j***			**Line y = mx+b from A19-A22**		**11** ***j***			**Line y = mx+b from F19-F22**
π fraction	x	y	$\Delta y/\Delta x$=m -0.44523		π fraction	x	y	$\Delta y/\Delta x$=m -0.4452
0.7333333	-0.66913	0.743145	y_0-mx_0=b 0.445229		0.7333333	-0.66913	0.743145	y_0-mx_0=b 0.44523
28 ***j***			**Line y = cx+d from A23-A28**		**25** ***j***			**Line y = cx+d from F23-F28**
π fraction	x	y	$\Delta y/\Delta x$=c -2.24604		π fraction	x	y	$\Delta y/\Delta x$=c -2.246
1.8666667	0.913545	-0.40674	y_0-cx_0=d 1.64512		1.6666667	0.5	-0.86603	y_0-cx_0=d 0.25699
6 ***j***			With 180° rotational		**9** ***j***			
π fraction	x	y	symmetry, each		π fraction	x	y	
0.4000000	0.309017	0.951057	side checks 2 pts.		0.6000000	-0.30902	0.951057	