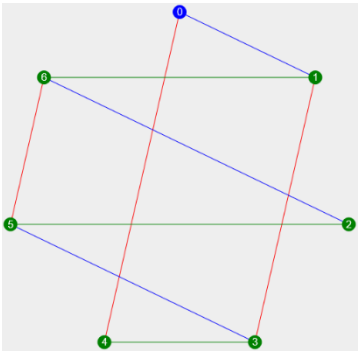
How Many Lines are in a Triangles Image?

Even before we start counting triangles we might wonder, *how many lines are in an image?* The rough answer is that there are ***n***/2 lines in each direction, or 3***n***/2 in a triangles image. In fact, *Lines* is one of four values 3***n***/2-3 ≤ *Lines* ≤3***n***/2 if ***n*** is even and 3(***n***-1)/2 if ***n*** is odd. Here we explore why this is true and we show why each of these answers occurs.

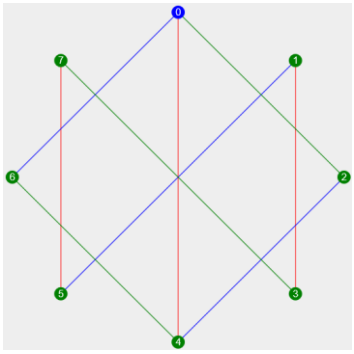
The Parallel Lines Drawing Rule*.** We start by defining a direction for a line to go using the vertices of the ***n-gon. From there we draw parallel lines on both sides of that line by adding one vertex at one end and subtracting one vertex from the other end until you cannot draw additional lines on that side. Since each line requires two distinct vertices, ***n***/2 lines are possible in a direction if all vertices are used. But sometimes not all vertices get used.

What happens when *n* is odd? If ***n*** is odd, there **must be** a vertex that is not paired with another vertex because that is the essence of being odd. Recall we defined an odd ***n***-gon as ***n*** = 2***k***+1. Since this happens in each direction, there are 3(***n***-1)/2 or 3***k*** lines in an odd ***n***-gon triangles image, so there are 3 lines each or 9 in the 7-gon image to the right. Notice in this image there are three vertices that have only two lines and the other four have three. Vertex 0 has no **green line** because there are two vertices between the line spanning **6-1** (or put another way, 0 is the vertex in the middle), so 0 is odd-vertex-out for **green**. Vertex 2 has no **red line** because there are two vertices between the line spanning **1-3**, so 2 is odd-vertex-out for **red**. Lastly, vertex 4 has no **blue line** because there are two vertices between the line spanning **3-5**, so 4 is odd-vertex-out for **blue**.

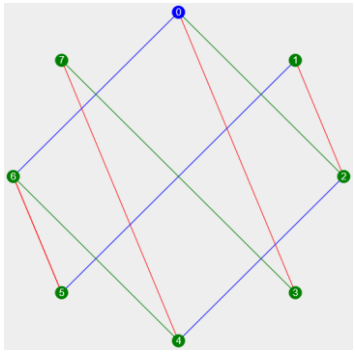


What happens when *n* is even? If ***n*** is even, things are not quite as cut and dried. An even ***n*** means that all vertices might be paired up, or there might be a left over vertex at each "side" for a given color (or direction). The four images below show the range of outcomes. The smallest number of lines of a given color is ***n***/2-1 and this occurs when the smallest line spans two vertices (like all three colors in the left image). The largest number of lines of a given color is ***n***/2 since a vertex is never attached to more than one line of that color because of *the parallel lines drawing rule*. This occurs when the smallest line of a given direction spans a single vertex (like all three colors in the right image).

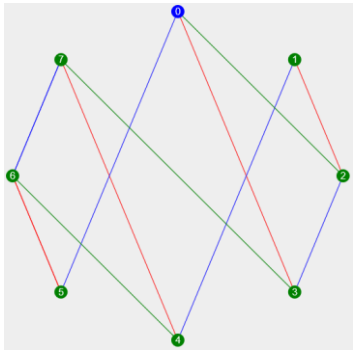
The number of lines in a triangles image with even ***n*** is between 3***n*** /2-3 and 3***n*** /2, or from 9 to 12 for ***n*** = 8.



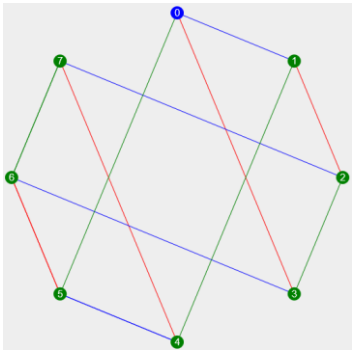
9 lines total **0-4**, **0-6**, **4-6**
6 vertices with 2 lines
2 vertices with 3 lines



10 lines total **0-3**, **0-6**, **4-6**
4 vertices with 2 lines
4 vertices with 3 lines



11 lines total **0-3**, **0-5**, **4-6**
2 vertices with 2 lines
6 vertices with 3 lines



12 lines total **0-3**, **0-1**, **1-4**
0 vertices with 2 lines
8 vertices with 3 lines

There is a single vertex or two vertices separating the endpoint vertices of the smallest line drawn. If there is a single vertex between then adding one and subtracting one simply replicates that last line. But if there are two vertices between the last line drawn, then one cannot draw a *non-degenerate* line from that in-between vertex to itself.

The three directions for parallel lines in the *General Triangles* file are described by four parameters, ***J***, ***K***, ***V***, and ***W***. We only need 4 because we know that at least 2 must include 0 so the directions are noted as **0-*J*** and **0-*K***. The last line may not include 0 so it is called ***V*-*W*** and values for the three directions are shown beneath each image. If ***n*** is even, there will be ***n***/2-1 lines if ***J*** is even and ***n***/2 lines if ***J*** is odd. The same is true for ***K***. If the sum ***V***+***W*** is even, there will be ***n***/2-1 lines and ***n***/2 lines if ***V***+***W*** is odd. These results are easily verified by comparing lines by color across the above images.