Testing for Concurrence using *Excel*

One of the most basic rules of geometry is that if two lines are not parallel, they intersect at one point. A natural extension is that three lines that are not parallel to one another will intersect at three points, and these three points determine a triangle. This is true UNLESS all three lines intersect one another at a single point. When this happens, we say that the lines are *concurrent*.

The *parallel lines* model sets up three directions for parallel lines using the vertices a regular ***n***-gon. By construction, all ***n***-gon vertices with three lines are *points of concurrency*. More interesting are the interior points where three lines appear to intersect at a single point. One can easily test for this in *Excel* using some basic trigonometry together with equations of lines, and equations for the intersection of those lines. The steps are laid out in the *Excel* file and will not be discussed at length here, but a couple of words are in order.

PwP versus plain vanilla trigonometry. The decision was made at the start of working on **PwP** and **ESA** to use an old-fashioned clock as a point of reference. As a result, vertices are enumerated clockwise, starting at the top, (x, y) = (0, 1), with the one exception that 12 is replaced with 0 so the ***n*** vertices are numbered from 0 to ***n***-1 (which are the only possible remainders once any number is divided by ***n***). By contrast, plain vanilla trigonometry starts at the right side (3 o'clock or (x, y) = (1, 0)) and numbers vertices counterclockwise from there.

Although trigonometric functions are exactly how the vertices of the ***n***-gon are placed in **PwP** and **ESA**, they are not laid out in the standard trigonometric fashion (if interested, the Polygon Vertices sheet of the General Triangles file lays this out in detail). Rather than work through the equations for x and y coordinates involved in correctly identifying x and y, simply know that the solution switches x and y coordinates. Any potential intersection in the quadrants I and III will maintain signs. And an intersection in the second quadrant in **PwP** will have (x, y) values from quadrant IV and *vice versa*. Take a simple example based on ***n*** = 12. Vertex 5, 5 o'clock, is in IV, with positive x and negative y. But if we calculate those coordinates using trigonometry as (cos(θ), sin(θ)) where θ = (2π)·5/12 the result is (x, y) = (-0.867, 0.5) in quadrant II. Effectively, in **PwP**, 5 o'clock switches those x and y coordinates and places 5 at (0.5, -0.867) in quadrant IV.

This graphic summarizes the differences discussed above. Trigonometric vertices are counted counterclockwise starting at (1, 0) and **PwP** vertices are counted clockwise starting at (0, 1). One is like looking sideways through a mirror at the other. This may sound coufusing but the main point to remember is ***just think of x as y and y as x***.

Playing with Polygons vs Trigonometry

(0, 1) Start for PwP — Clockwise

Quadrant II | **Quadrant I**

Counter-clockwise — **(1, 0) Start for Trigonometry**

	Fraction of vertices	Numbered small to large	Coordinates x	y	Fraction of vertices	Numbered small to large	Coordinates* x	y	
Trigonometry	**0.25/*n* to 0.5/*n***	**V axis to H**	**x -**	**y +**	**0 to 0.25/*n***	**H axis to V**	**x +**	**y +**	**Trig.**
PwP	**0.75/*n* to 1**	**H axis to V**	**y from IV**	**x**	**0 to 0.25/*n***	**V axis to H**	**y**	**x**	**PwP**
Trigonometry	**0.5/*n* to 0.75/*n***	**H axis to V**	**x -**	**y -**	**0.75/*n* to 1**	**V axis to H**	**x +**	**y -**	**Trig.**
PwP	**0.5/*n* to 0.75/*n***	**V axis to H**	**y**	**x**	**0.25/*n* to 0.5/*n***	**H axis to V**	**y from II**	**x**	**PwP**

Quadrant III | **Quadrant IV**

* Signs of x and y noted after trigonmetry coordinates.

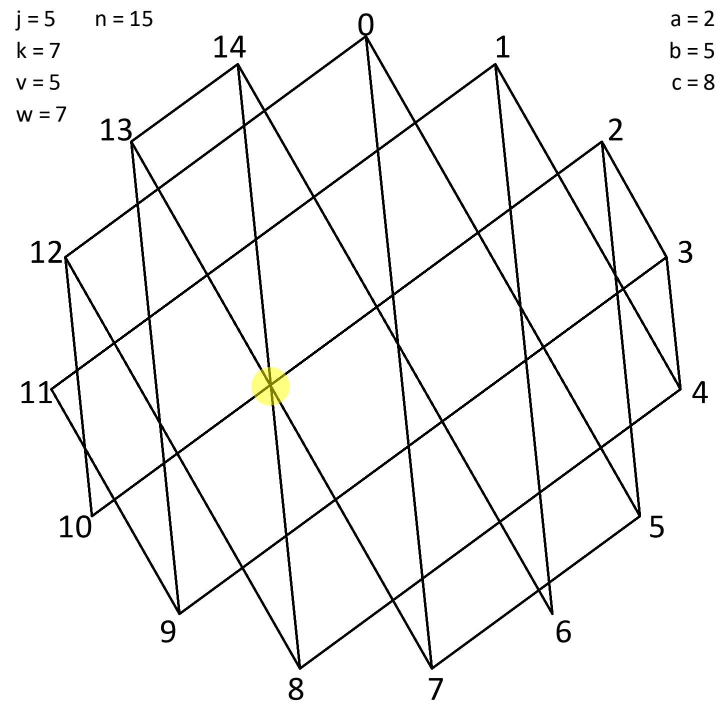
The Concurrence *Excel* file. Various pages of the Testing for Concurrence file are provided. One, used in discussing equilateral triangles examines two different images with each image having horizontal lines. Another version allows testing two points on the same image by tying together certain cells. A third relaxes the horizontal lines assumption and allows for testing given three lines in different, non-horizontal, directions. Rather than show each here, we will see screenshots of these pages elsewhere within **PwP** as the need arises.

An Example. The example seen here answers the question posed in the 0-5-7 triangles images discussion of whether there is concurrence in this image at the highlighted point. This intersection is in the third quadrant with x coordinate larger than y coordinate and both negative.

The description at the top of the table is included here (and has been discussed above) but the main part of the table shows the answer. This is not a point of concurrence. Two of the three intersection points are found in the table.
The coordinates of the 2-10 and 14-8 intersection is:
(-0.3013, -0.0897).
The coordinates of the 2-10 and 13-7 intersection is:
(-0.3039, -0.0916).

Since these are not equal, the lines are not concurrent.

Change **A19** from 2 to 13 and **A22** from 10 to 7 to find the intersection of 13-7 and 14-8 which is (-0.3005, -0.0976). This is the point of a small triangle just below its 2-10 base.



Concurrent Points 3 lines are concurrent when they pass through one another at a common point. This point is called the point of concurrency.

Since this analysis uses trigonometry, vertex 0 is at (1,0) and vertices are numerated counterclockwise.
For example, here are vertices 6, 12, 18 around the 24-gon. Vertex numbering starting ↶ from (1,0).

24 ***n***

j	π fraction	x	y
0	0	1	0
6	0.5	6.13E-17	1
12	1	-1	1.23E-16
18	1.5	-1.8E-16	-1

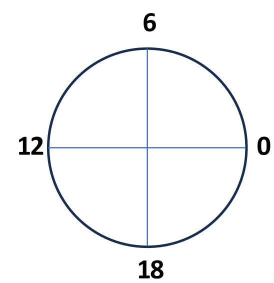
***Equations for* (x, y)**

$\pi_{fraction} = 2*j/n$

x = COS($\pi_{fraction}$*pi())

y = SIN($\pi_{fraction}$*pi())

where pi() = 3.1415...



So, x and y are switched and order of counting is reversed relative to PwP.

This examines both apparent concurrent points on an ***n*** = 15 image.

0 *j₀*			j_0-(n-j_0) is a horizontal line in the ***n***-gon, vertical here.	
15 ***n***				
π fraction	x	y	Solve for x in	
0.0000000	1	0	mx+b = cx+d	
			x* = (d-b)/(m-c)	
			x*	**-0.0897**
2 *j*			Check: Same y at x*.	
π fraction	x	y	mx+b	**-0.30129**
0.2666667	0.669131	0.743145	cx+d	**-0.30129**
10 *j*			**Line y = mx+b from A19-A22**	
π fraction	x	y	Δy/Δx=m	1.376382
1.3333333	-0.5	-0.86603	y_0-mx_0=b	-0.17783
14 *j*			**Line y = cx+d from A23-A28**	
π fraction	x	y	Δy/Δx=c	-0.1051
1.8666667	0.913545	-0.40674	y_0-cx_0=d	-0.31072
8 *j*				
π fraction	x	y		
1.0666667	-0.978148	-0.20791		

If equal, then concurrent point

0 *j₀*			j_0-(n-j_0) is a horizontal line in the ***n***-gon, vertical here.	
15 ***n***				
π fraction	x	y	Solve for x in	
0.0000000	1	0	mx+b = cx+d	
			x* = (d-b)/(m-c)	
			x*	**-0.09161**
2 *j*			Check: Same y at x*.	
π fraction	x	y	mx+b	**-0.30393**
0.2666667	0.669131	0.743145	cx+d	**-0.30393**
10 *j*			**Line y = mx+b from F19-F22**	
π fraction	x	y	Δy/Δx=m	1.37638
1.3333333	-0.5	-0.866025	y_0-mx_0=b	-0.1778
13 *j*			**Line y = cx+d from F23-F28**	
π fraction	x	y	Δy/Δx=c	-0.5774
1.7333333	0.669131	-0.74314	y_0-cx_0=d	-0.3568
7 *j*				
π fraction	x	y		
0.9333333	-0.97815	0.207912		