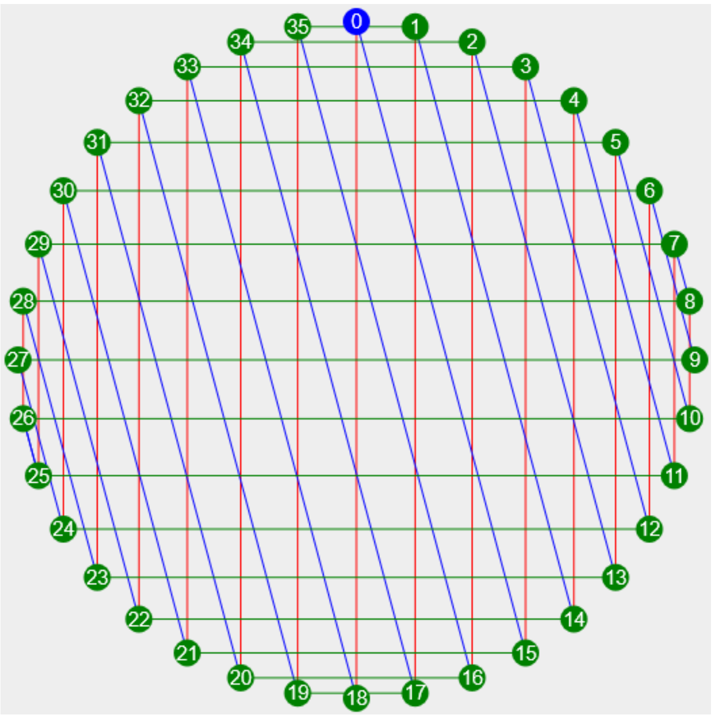
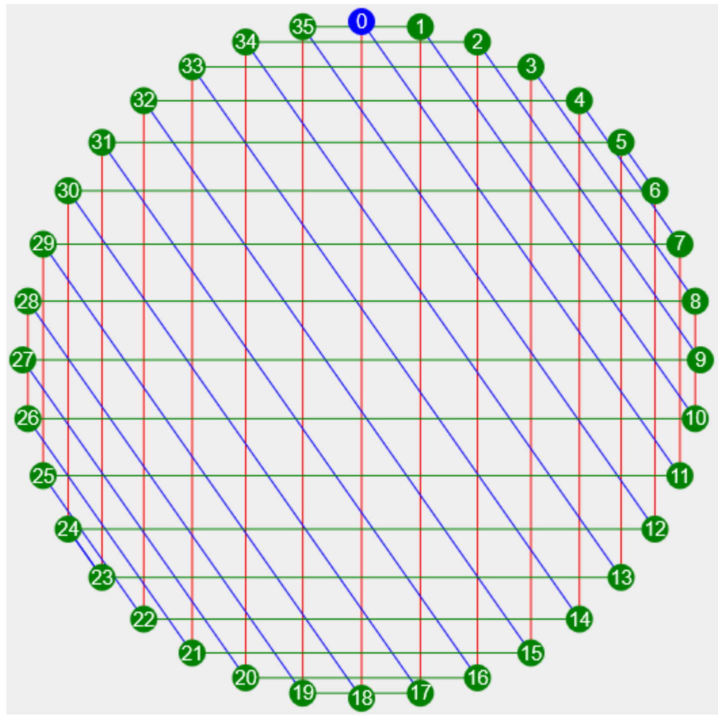
Using the Web Version to Understand why there are a-1 Internal Arcs of an a, b, c Image

The angles created in the *General Triangles* model span a, b, and c vertices, where a is the smallest span and $a+b+c = n$. We examine counting patterns using the sharpest angle a, the apex, as the *distinguished point* of a triangle. Chapters 3 and 4 restricted the analysis to $a = 1$ so that these sharp points of any triangle are exclusively located at the vertices of the n-gon. In Chapter 5, when $a = 2$, we see that there are now internal apexes that must be counted as well as apexes at vertices. In our discussion of 0-5-7 images we asserted that, in general, there will be a-1 arcs of internal apexes. We examine that assertion here. To make the discussion as simple as possible, $n = 36$ so that a single spanned vertex is 5°, with **green horizontal**, **red vertical**, and **blue diagonal** lines. This results in right triangles images; two are shown here.





Apexes are red-blue intersections. Both images have lots of intersections, but for our purposes, ignore the intersections involving **green** lines as these are the horizontal bases of the triangles that we use for counting and focus instead on the **red**-**blue** intersections because these are the intersections that produce sharpest angle interior apexes. It should feel like the left image has a lot fewer **red**-**blue** intersections than the right image, and it does. The left image was chosen because it is easy to see that there are two arcs of interior apexes. The upper arc starts between vertex 7 and 8 at the intersection of **7-11** and **6-9** approaching concurrence with the **7-29** horizontal line (but tests show none are concurrent) and ending close to vertex 27 at the intersection of **26-28** and **24-27**. The lower arc starts near 9 at the **8-10** and **6-9** intersection, approaching concurrence with the horizontal line at **11-25** and ending between vertex 25 and 26 at the intersection of **25-29** and **24-27**. [*Two asides:* (1) It helps to *Toggle Vertices* on and off to see the intersections near the sides. (2) Given 180° rotational symmetry it is not surprising that the lower arc ending is (n-starting) of the upper arc, and (n-ending) of the upper arc becomes starting values for the lower arc. To take 1 of 4 examples to help explain this observation: the first vertical **7-11** mentioned in the upper arc and the last lower arc vertical **25-29** are rotationally identical since 29+7 = 36 as is 25+11.] The right image has more **red**-**blue** intersections, but it is harder to see the arcs.

Focus on the Legs of the Apex Angle at 0. The angle **15**-0-**18** spans 3 vertices, $a = 3$, and creates 15-75-90° triangles. An even quicker way to see that this span is 3 is to look up along the vertical line **15-3** from 15 to 3. The angle **0**-15-**3** also spans 3 vertices. Now, **focus on the leg common to both angles, 0-15**. Notice that there are two **vertical lines** between **0-18** and **3-15**, namely the ones starting at 1 and 2. The **red**-**blue** intersection of the vertical line **1-17**, is an apex on the upper arc and **2-16** is on the lower arc. Following the same logic on the right image, the angle **0**-11-**7** spans $a = 7$ vertices and creates 35-55-90° triangles. This image has a-1 or 6 internal arcs since the line **0-11** intersects **vertical lines** starting at 1 through 6 on the interior of the image. Other **blue diagonals** have **6 red** internal intersections until near the edge in both directions. In the image at right, the first **blue lines** with fewer intersections are **3-8** and n/2 more, or **21-26**.