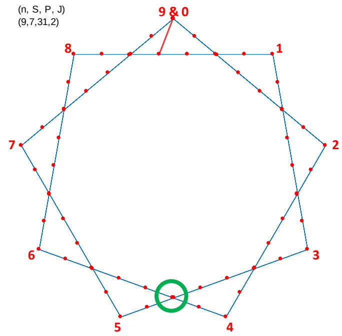
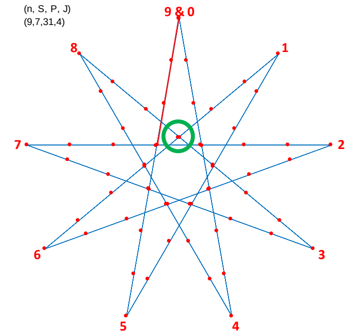
# 1.1.2. A Comparative Introduction, Part II

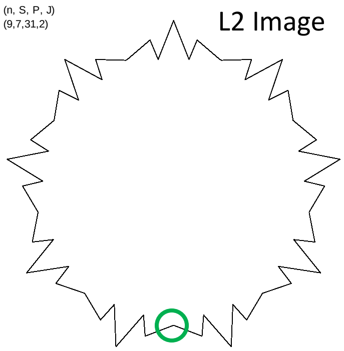



Left 2: L2 **frame** and **dots**

The second part of the introductory section focuses attention on what we can do once we have some base images to work with. Section 1.1.1 produced four images. The first two were called *porcupine polygons* because they were very prickly and they are easy to create by going almost "half-way" around the polygon before drawing the first line, without being exactly half-way around. (If you draw a line exactly half-way around, the end result is a pretty uninteresting vertical line.) One polygon had 9 sides, the other had 6.

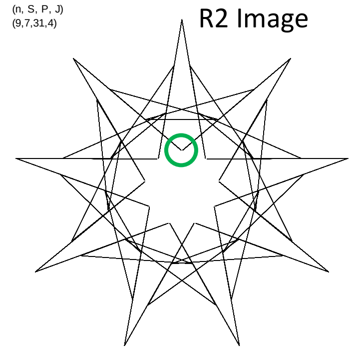



Right 2: R2 **frame** and **dots**




L2 Image

Next, we created images using the same ***n*** = 9, ***S*** = 7, and ***P*** = 31 values that produced the *porcupine 9-gon* with one adjustment. Instead of jumping to the next vertex, we jumped 2 or 4 vertices to produce the **9,2-star** or **9,4-star frames** shown at top left (L2) and right (R2). In each instance, the **first line** is 3 subdivisions in on the 5th line of the vertex frame because 31 = 4·7+3. The first 5 **frame lines** of L2 are **0–2–4–6–8–1** and for R2 are **0–4–8–3–7–2** (**–** means draw a **line**). The first line of the image is **a red line from the top to the third subdivision dot** on the **8–1 frame line** for L2 and the **7–2 frame line** for R2.




R2 Image

(Once again, you are advised to **focus on *images*,** not *explanations,* **in your initial reading** of Sections 1.1.1 and 1.1.2.)

The discussion above dives deeper into how the **vertex frame star** was created and how the **first line of the image** uses **subdivision dots** placed on the **frame**, but the final images are the same here as on page 2 of 1.1.1 (which is why we continued the labelling from 1.1.1 and call them L2 and R2). Quite clearly, the prickly porcupine nature of images when ***J*** = 1 and ***P*** is close to ***n·S***/2 (half of all subdivisions) no longer holds if ***J*** > 1 as the L2 and R2 final images attest.

**ESA allows you to quickly answer questions and test new ideas.** Suppose you liked those prickly images. You might ask:

*Is it possible to adjust **P** and create images that are prickly when **J** > 1?* ***There are two ways to obtain an answer:***

***1. Play to Learn.*** We could simply adjust ***P*** using the ⬍ arrows. That is, we could proceed by simply *playing* with ***P***.

***Playing* with this book.** The term *playing* is used advisedly here. Specifically, it means working with the files (in *Excel* or via the web version) to discover what you find interesting. Think of playing as exploring. The book then encourages the reader to think about why it is interesting and *play* some more. As you *explore*, you may begin to see patterns in those explorations. That is, you may begin to *extract* patterns from your explorations. Finally, you may be able to *explain* what you have found. Sherman Stein called this *The Triex: Explore, Extract, Explain*. He argued that this is the essence of more deeply understanding mathematics and it is also provides a useful starting point for how to teach mathematics.

***2. A more analytical approach.*** We could use what we learned from the **nail** locations in images L2 and R2. What if we choose a ***P*** that is associated with one of the **green circled nails**? The first line would then be almost vertical. Finding that ***P*** is really quite simple. To see how, look at the L2 and R2 **frame** and **dots** images above.

***P*** for the L3 image. The 9,2-star in L2 starts at the top then jumps two vertices each time: **0-2-4-6**. The **circled dot** is the 2nd **dot** after vertex **4** on the way to vertex **6**, so ***P*** = 7+7+2 = 16.

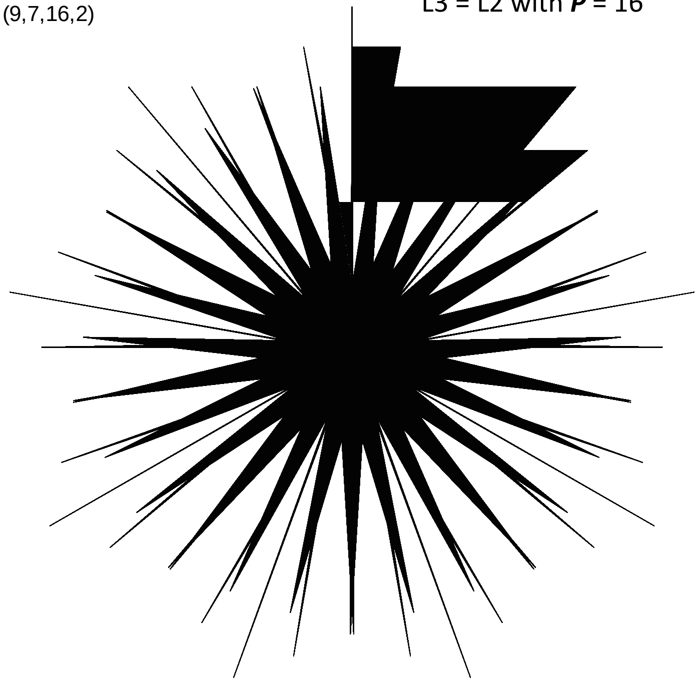
***P*** for the R3 image. The 9,4-star in R2 starts at the top then jumps four vertices each time: **0-4-8-3**. The **circled dot** is the 3rd **dot** after vertex **8** on the way to vertex **3**, so ***P*** = 7+7+3 = 17.

The resulting images are instructive. You can see the **9,2-star** in the pattern of needlepoints in the L3 image and notice that the second needle in from each vertex needle (the longest ones) is in fact two needles, just as we expect given our **circled dots** observation in L2. The R3 image has 9 vertex needles with each needle open to the center and the rest of the image creating a superstructure surrounding those needles. Both images are also created using 63 connected lines.

The 8 images beneath L3 and R3 show *some* results from the ***Play to Learn*** approach given ***J*** = 4 (R2 **frame** with ***P*** noted).

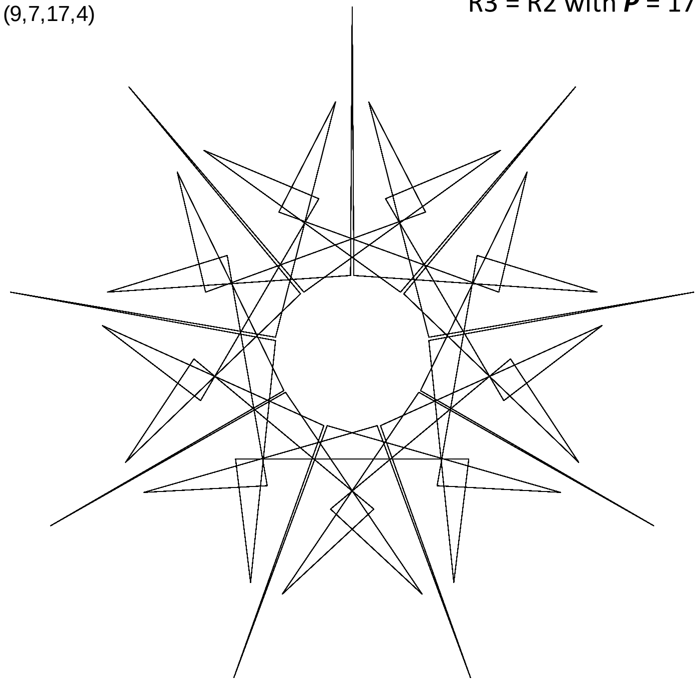
(n, S, P, J)
(9,7,16,2)

L3 = L2 with ***P*** = 16

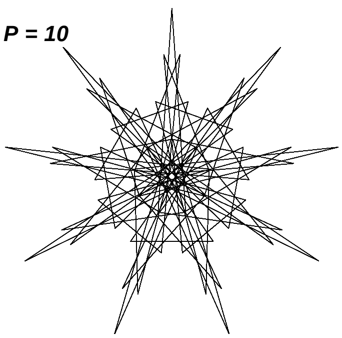


(n, S, P, J)
(9,7,17,4)

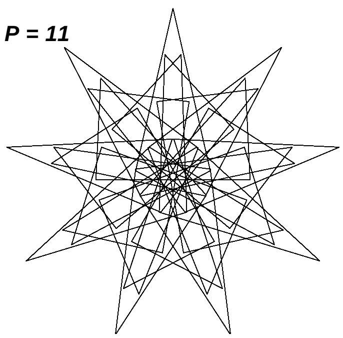
R3 = R2 with ***P*** = 17



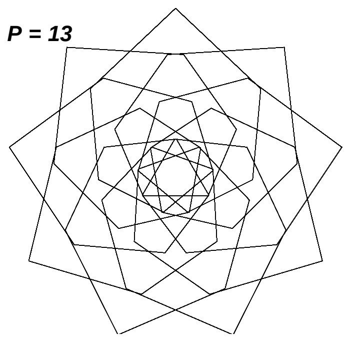
***P = 10***



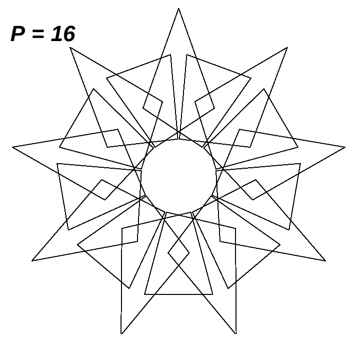
***P = 11***



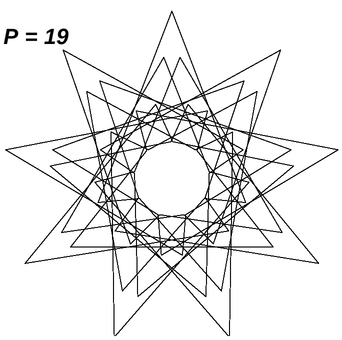
***P = 13***



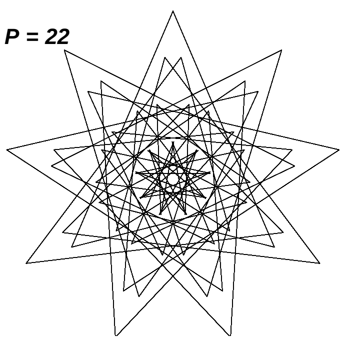
***P = 16***



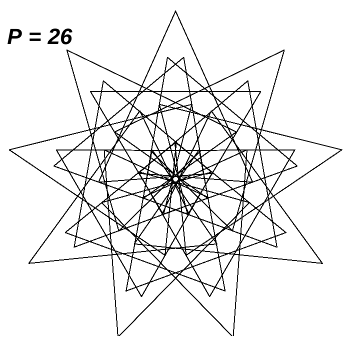
***P = 19***



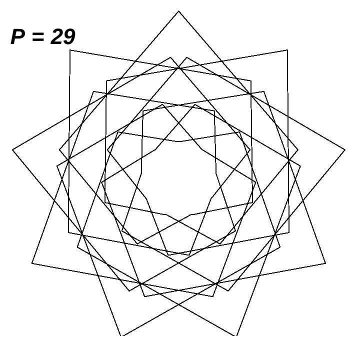
***P = 22***



***P = 26***



***P = 29***



**NOTE:** *The **14 distinct final images** from Sections 1.1.1 and 1.1.2 would have taken **days** to create via traditional methods, not **minutes** using ESA.*

**What is in the rest of this chapter?** This book is meant to be interactive in nature and was not written to be read from front to back. Section 1.2 provides a rationale for this somewhat unusual statement. Therefore, the book is written as a series of *explainers* (using the third *ex* of Stein's *Triex*) – short, targeted sections that need not be read sequentially, beyond the few ***Just the Basics*** sections noted in 1.2. Think of *explainers* as short, self-contained vignettes that are digestible in a single sitting.

Section 1.3 discusses the layered nature of the mathematics involved in ESA and notes that one can enjoy the images while not completely understanding how they were created. Section 1.4 provides an overview of the book as a whole.