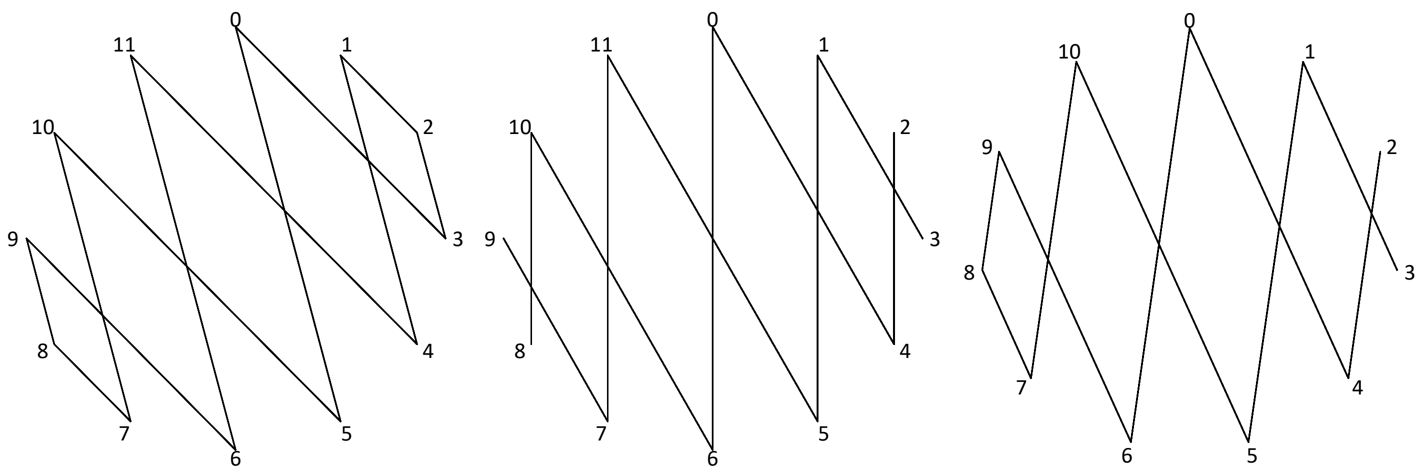
An Introduction to Second Sharpest Images

In the previous two chapters we focused attention on sharpest triangles images for a simple reason. If the sharpest angle (the apex) spans a single vertex, then all triangle apexes must occur at vertices of the ***n***-gon. In this setting, we only needed to examine apex counts on the perimeter of the ***n***-gon. In this chapter we examine images in which the sharpest angle spans two vertices. Second sharpest images have interior apexes aligned on a diameter of the circle containing the ***n***-gon vertices as well as apexes at those vertices.

Second Sharpest Internal Intersections are on a Diameter of the Circle. It is straightforward to see why second sharpest images have internal intersections on a diameter of the circle; we need only look at how the two lines that create the second-sharpest angle intersect. This is easiest to see if the third line is suppressed because then the only lines visible are those producing the second-sharpest apex angles. The three images below cover the stylistic possibilities for second sharpest apex angles. The first two are based on ***n*** = 12, and the third is ***n*** = 11.



Consider the angle created at 0 and ending at ***J*** and ***K***. Since the smallest angle spans two vertices, either both are even, or both are odd. The left image shows odd ***J*** and ***K*** for even ***n***, in which case the ends have diamonds (like at 2 and 8 = 2+***n***/2), and the middle shows even ***J*** and ***K*** for even ***n*** with obtuse diamond ends at internal intersections (like at 2.5 and 8.5= 2.5+***n***/2). Other choices where |***J***-***K***| = 2 will simply be rotated version of these images if ***n*** does not change. When ***n*** is odd, like the right image, one end will be the obtuse angle of a diamond (like at 8) and the other shows an internal obtuse angle intersection (like at 2.5 = 8-***n***/2). Any other even or odd ***n*** will have apex lines intersection endpoints that look the same as the above images. The only difference is that there will be more or fewer diamonds side-by-side (and their apexes will be sharper or less sharp). Because the ***n***-gons are regular, each of the obtuse corners of a diamond are on a diameter created by these endpoints, and this diameter is a line of symmetry.

It is important to recognize that although it is a line of symmetry with respect to these two lines, it will not in general be a line of symmetry for the overall image. (If this statement does not make sense, image what a horizontal third line would do to each image. Clearly the resulting triangles images would not be symmetric across the diameters noted above, even though the lines creating the apex angles are symmetric across those diameters.) This is why the third line has been suppressed in the images above.

Chapter Outline. Rather than provide an exhaustive examination of second sharpest images like was done for sharpest images in the past two chapter, we follow a different path here. To model how to undertake your own analysis, this chapter starts by showing a complete *worked example* for a class of images. The worked example examines second sharpest odd acute scalene triangles, the closest neighbor to sharpest odd isosceles triangles from Chapter 3. The next example is only partially worked but uses the same ***JKVW*** functional relations in the worked example, this time on even polygons. The resulting triangles are second sharpest scalene right triangles, the closest neighbor to sharpest even right triangles from Chapter 4. Additional sections are devoted to what happens when a fixed set of vertices are chosen, and ***n*** is allowed to vary. Finally, we end with a bookend to the sharpest odd isosceles triangles analysis of Chapter 3 by examining sharpest even isosceles triangles images as challenge questions. An *Excel* file dedicated to sharpest isosceles triangles for any ***n*** is provided to streamline this analysis.