Angles of Triangles in *General Triangles* Images

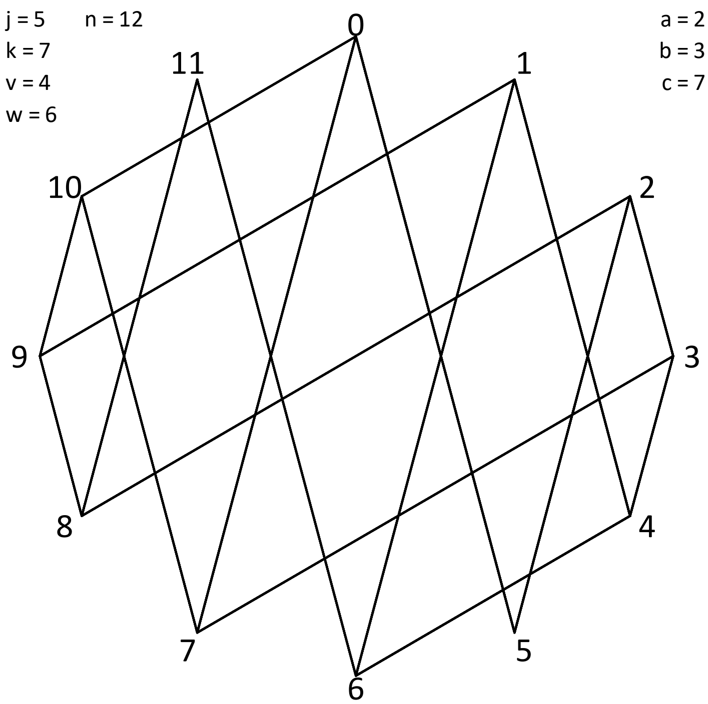
The *General Triangles* model uses five parameters, ***n***, and ***j***, ***k***, ***v***, and ***w***. These produce sets of parallel lines in three directions and hence triangular images. Triangular images means that there must be 3 angles summing to 180°, although some of the angles may be the same. These angles, based on vertices of the ***n***-gon spanned, must be multiples of 180/***n***° and they must sum to 180°. Given ***n*** = 12, angles mut be multiples of 15°. Here we explore how angles are obtained from these parameters. To focus the discussion, both images maintain ***v*** and increase ***j***, ***k***, and ***w*** by 1 to maintain ***a***, ***b***, and ***c***.

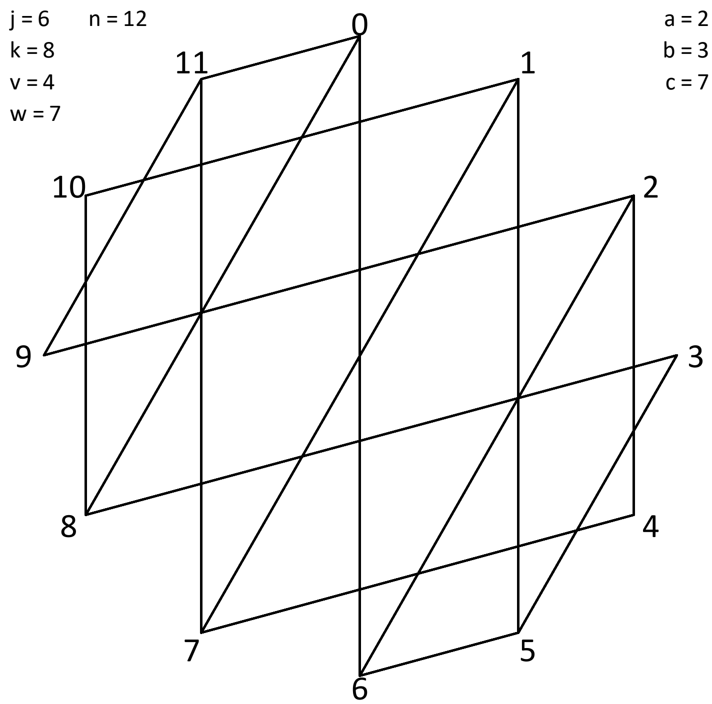
Visualizing *a*, *b*, and *c*. Even though both images were chosen because their parameters do not directly delineate a triangle where all three corners are vertices of the ***n***-gon (as is the case when ***v*** = ***j*** and ***w*** = ***k***), the left image has one such triangle, 1-4-6. By contrast, the right image has no such triangle, despite having the same angle spans ***a***, ***b***, and ***c***.

Left image: Sometimes the spanned vertices are readily visible like the left image because there exists a triangle in which all three corners are vertices of the ***n***-gon. This makes it quite clear what the angles are. The sharpest, angle 1-4-6 spans ***a*** = 2 two vertices (6-4) or 30° = 2/12·180°. The middle-sized angle 1-6-4 spans ***b*** = 3 vertices (4-1) or 45° = 3/12·180°. The largest angle 1-4-6 spans ***c*** = 7 vertices (from 6 to 1) or 105° = 7/12·180°. Cumulatively, ***a***, ***b***, and ***c*** span the whole circle.

Right image: No triangle in the right image has all three corners that are vertices of the 12-gon. Nonetheless, the three angles are clear. The smallest angle ***a*** spans 2 vertices (like 6-0-8), the middle-sized angle ***b*** spans 3 vertices (like 0-8-3) and the largest angle ***c*** spans 7 vertices (like 0-11-7). Cumulatively, ***a***, ***b***, and ***c*** span the whole circle and sum to ***n***. In discussing these angles, we could have focused on a single triangle like we did with the left image by using the interior angle theorem instead of relying on the inscribed angle theorem for implicit guidance.

Degrees versus vertices spanned. Note that instead of talking in degrees, we can talk in terms of spanned vertices since ***a***+***b***+***c*** = ***n***, here 12, because (***a***+***b***+***c***)/***n***·180° = 180°. It is often easier to talk about vertices spanned rather than degrees.





Calculating *a*, *b*, and *c*. This screenshot from the *General Triangles* file based on the right image shows the equations used to find ***a***, ***b***, and ***c*** from ***j***, ***k***, ***v***, ***w***, and ***n***. These values are based on the absolute value of differences between various parameters. By construction, ***a***, ***b***, and ***c*** are ordered from smallest to largest given these equations.

a , ***b*** , ***c*** are arcs of circle summing to ***n*** and represent angles 180*a*/*n* °, 180*b*/*n* °, and 180*c*/*n* °.		2 = ***a*** = min(*a'*,*b'*,*c'*)
Steps to obtain	11 = *s* , The line *vw* is parallel to 0*s* with *s* = MOD(*w* +*v* ,*n*)	3 = ***b*** = ***n*** - ***a*** - ***c***
a ,*b* ,and c from	2 = *a'* = min(|*j*-*k* |,|*j*-*s* |,|*s*-*k* |) Note: *a', b'* and *c'* are unordered.	7 = ***c*** = MAX(*a'*,*b'*,*c'*)
j ,*k* ,*v* ,*w* ,& *n* :	3 = *b'* = MAX(|*j*-*k* |,|*j*-*s* |,|*s*-*k* |) - *a'* 7 = *c'* = *n* - *b'* - *a'*	With ***a*** ≤ ***b*** ≤ ***c*** and ***a*** +***b*** +***c*** = ***n*** .