1.3. The Layered Nature of the Mathematics in ESA

A natural question is: ***How much math do you need to know to enjoy this book?***

The interesting answer is this: ***That depends on how you want to enjoy this book.***

ESA would not be possible without the electronic files that create the string art images, and those files can be manipulated by users without any mathematical training at all simply by changing parameters with ⬍ arrows. Users will inevitably learn things about the images as they play even if what they learn is informal in nature. Users learn that you see things happening with even numbers that differs from odd numbers even if those notions have not yet been formally defined in an educational setting. Stein argues that such informal learning cements formal understanding once that takes place.

Users are willing to play with the files because the images they create are engaging. They can create their own beautiful images, and then change those images just by pointing and clicking. Users learn more as they search for other similar images, finding out what works and what does not as they change parameters, deepening their understanding of mathematics even if that is simply the side-effect of this play.

The book's value-added is that it provides a comprehensive guide which acts as a road-map exploring why images change as parameters change or are drawn the way they are drawn for a fixed set of parameters. The question therefore becomes: ***How much math do you need to know to understand what is going on with the images?***

What mathematics is required? The basics can be fully understood even without knowledge of multiplication and division since lines are simply placed connecting every $\boldsymbol{P}^{th}$ endpoint, so one need only be able to count to ***P*** over and over again (if one were manually counting). Of course, repeated addition is the edge of multiplication, and the location of ***P*** on the frame created by the ***n***,***J***-star is easiest to explain using division. The *Guided Inquiries* in Section 25.5 walks younger users through seeing the effect of commonality between parameters without requiring them to know about the concept of greatest common divisor. (A more formal examination of commonality is provided in Chapter 21 which includes discussion of a method of finding the GCD of two numbers that is more than 2000 years old in Section 21.2.3, *Euclid's Algorithm*.) Despite this low mathematical bar, the book will be of interest to those with a greater degree of mathematical sophistication due to the strong visual appeal of the images created using just four numbers: ***n***, ***S***, ***P***, and ***J***.

The mathematics used is not advanced, but it is used intensively. The only mathematical concepts contained in the book beyond middle-school mathematics are a couple of theorems about angles from geometry, discussed in Chapter 22, and the idea of modular arithmetic, examined in Chapters 23 and 24. However, modular arithmetic is just a fancy term for doing old-fashioned division and ignoring everything except the remainder. If this sounds abstract, realize that you already know some modular arithmetic because you know that 5 hours later than 10 pm is 3 am and not 15 pm. Without thinking about it you threw away the 12 and kept the remainder of 3 when you added 5 to 10 and got 15. Clock arithmetic is a form of modular arithmetic, and this analogy will be used throughout the book, but especially in Chapter 17, *Four-Color Clock Arithmetic*.

Layered mathematics. This material can be considered from a layered perspective and examined with different degrees of mathematical sophistication. As a result, parts of some sections (or indeed, entire sections) are noted as **MA** (for **M**athematical **A**pproach). **MA** material can be skimmed or ignored by those who do not want to wade through the mathematics (such as that of modular multiplicative inverses, the topic of Chapter 24). And note that, even if an image requires an **MA** explanation to fully understand WHY it works, the image itself is intrinsically engaging as an artistic object. The *Teaching Companion to the Web Guide* in Section 26.4 provides examples of how you might layer your discussion of this material, even in an introductory setting.

Who should be reading (or using) this book? The book is primarily a book of recreational math. It could also be thought of as a recreational art book because it brings together math and art. The math is overt, the art is undeniable.

There are many ways in which this could be used in a classroom or in after-school settings for students in K-12. ESA is surprisingly flexible in covering topics at a wide range of class levels. Chapter 26 provides suggestions for teachers and there are ancillary teacher resources including links to Common Core State Standards on the ESA website.

This flexibility extends to college-level classrooms since this material provides geometric interpretations of topics covered in Number Theory and Abstract Algebra classes (like modular multiplicative inverses or group theory). Finally, there are links between this material and the literature on regular convex polytopes, see Coxeter 1974, but those connections have not been pursued in this book.