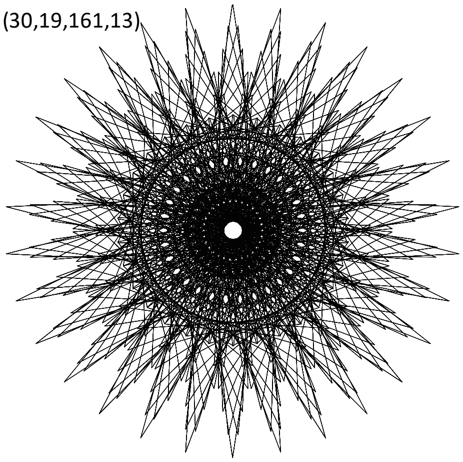
How are Nearby *P* Images Drawn Relative to *Single-Step P* values?

As a general rule, ***P*** values near a *single-step* ***P*** value do not have the systematically drawn character of the *single-step* image. Nearby ***P*** with SCF = 1 often look similar to the static *single-step* image but are not typically seen drawn in a *single-step* fashion.

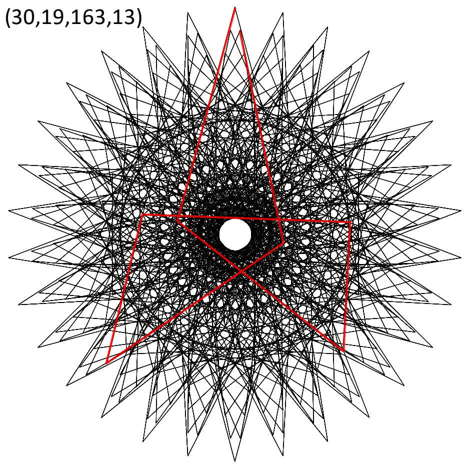
However, every ***P*** that is relatively prime to ***n·S*** has a modular multiplicative inverse ***M*** (***M·P*** = 1 mod ***n·S***), and the length of the *single-step* is ***L*** = minimum(***M***, ***n·S-M***) < ***n·S***/2 meaning that a subdivision endpoint within 1 of the top (the requirement for *single-step*) will occur in less than half the number of lines in the completed image. If this is near half the number of lines in the image then the final image is created in just a couple of these single steps.

The essence of single-step images is that you can see the pattern repeat in a small enough number of steps that the image looks systematically created and the focus of this chapter has been on ***L*** = 7. Here we broaden our horizon and consider nearby ***P*** values. We focus on two versions of 3SST, the 570-line original version (30,19,163,13), top row, and the more harmonious 253-line version from the previous section, (11,23,36,5) bottom row. The nearest coprime ***P*** values on both sides to each are shown at left and right.

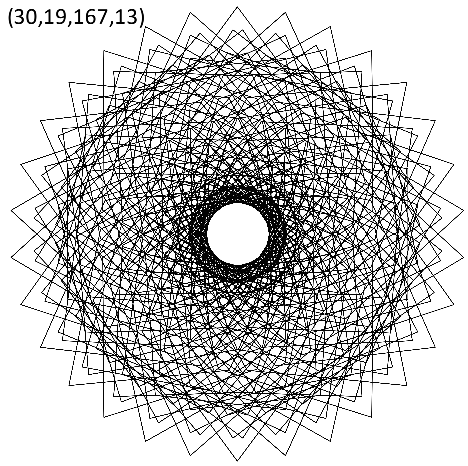
(30,19,161,13)



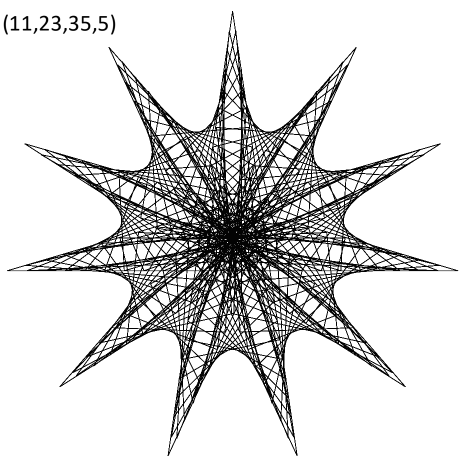
(30,19,163,13)



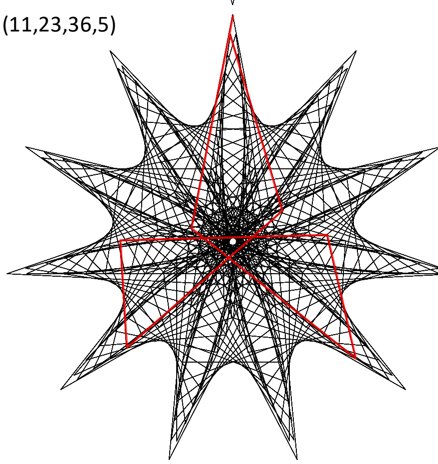
(30,19,167,13)



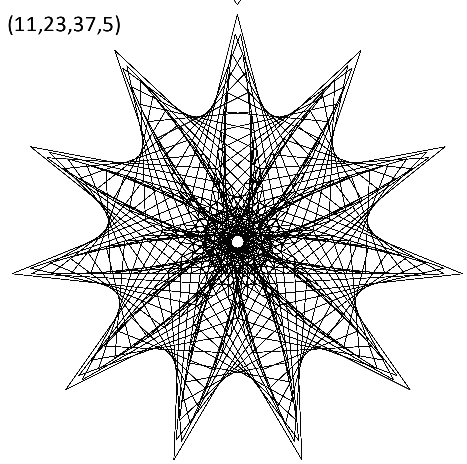
(11,23,35,5)



(11,23,36,5)



(11,23,37,5)



The first thing to notice is that the top right image does not appear similar to the other two top row images but all three bottom row images appear very similar. The parabolic curves are larger as P increases but otherwise, they appear to be morphed versions of one another. The reason for this is simple: since 570 has prime factors 2, 3, 5, and 19, only 31.6% of ***P*** values (180 of 570) are coprime to ***n·S***. This means that ***P*** values are more spread out (for reasons discussed here). By contrast, 253 has prime factors 11 and 23 so 87% of ***P*** values (220 of 253) are coprime to ***n·S***.

Four out of fifteen consecutive ***P*** values have SCF = 1 in the top row (1/2·2/3·4/5 = 4/15) unless one of those values is also a multiple of 19 in which case there are only 3 out of 15 coprime ***P*** to 570. By contrast, 10 out of 11 consecutive P values are coprime to 253 unless one of those is also a multiple of 23. This means that full-density images morph much more smoothly for the bottom row than the top.

The top portion of the table, A, shows *single-step* ***P*** values for lengths ***L*** from 2 to 25. 25 was chosen as the outer bound because each of these values of ***L*** means that the image is completed in at least 10 single steps given 253-line images.

The second portion, B, examines the 10 ***L*** values associated with ***P*** between 33 and 44 given 253-line images. The boxed area, C, focuses on 570-line images. MMI values are obtained by backtracking Euclid's Algorithm as discussed in E24.3 and negative MMI values equal ***n·S-M***, and the single-step ***P*** value is the smaller of these two values. The *single-step* ends one subdivision shy of the top, -1, if the negative MMI is smaller, and just after the top, 1, if the MMI is smaller.

A. *Single-Step* ***P*** values given lengths ***L*** with ***n · S*** = 11·23 = 253 for ***L*** from 2 to 25

Single-Step ***L***	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	21	24	25
MMI ***M***	127	169	190	152	211	217	95	225	76	232	39	235	135	174	134	239	40	38	241	116	81
-MMI, ***nS-M***	126	84	63	101	42	36	158	28	177	21	214	18	118	79	119	14	213	215	12	137	172
Single-step, ***P***	126	84	63	101	42	36	95	28	76	21	39	18	118	79	119	14	40	38	12	116	81
Ends at -1 or 1	-1	-1	-1	-1	-1	-1	1	-1	1	-1	1	-1	-1	-1	-1	-1	1	1	-1	1	1

B. *Single-Step* lengths ***L*** for ***P*** values between 33 < ***P*** < 44

P	34	35	36	37	38	39	40	41	42	43
MMI ***M***	67	94	246	212	20	13	19	216	247	153
-MMI, nS-M	186	159	7	41	233	240	234	37	6	100
L	67	94	7	41	20	13	19	37	6	100
Ends at -1 or 1	1	1	-1	-1	1	1	1	-1	-1	-1

C. Smallest *Single-Step* ***L*** given ***n · S*** = 30·19 = 570

Single-Step ***L***	7	11	13	17	23	29	31
MMI ***M***	163	311	307	503	347	59	331
-MMI, ***nS-M***	407	259	263	67	223	511	239
Single-step, ***P***	163	259	263	67	223	59	239
Ends at -1 or 1	1	-1	-1	-1	-1	1	-1

L values near ***P*** = 163

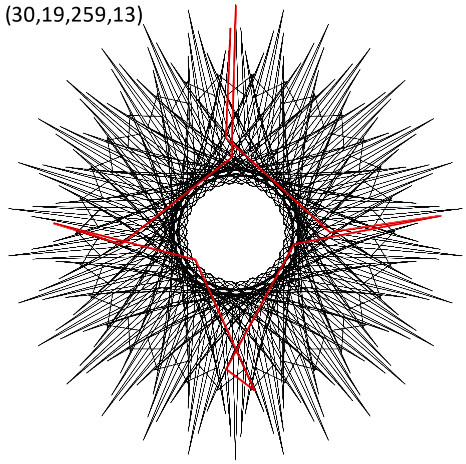
P	161	163	167
MMI ***M***	131	7	413
n · S - M	439	563	157
L	131	7	157
-1 or 1	1	1	-1

The highlighted ***P***, ***L*** pairs in B suggest that half of the curved-in 253-line cobwebs between 33 < ***P*** < 44 are completed in at least 12 single steps (12.7 = 253/20). Viewing the first ***L*** lines for each of these ***P*** values produces similar static images with very different single-step sub-images. These images include (from smallest ***L*** to largest): *P* = 42, DL = 6 is *spinning hexagons* (6·42 = -1 mod 253); *P* = 36, DL = 7 is *three shape-shifting triangles* (7·36 = -1 mod 253); *P* = 39, DL = 13, is *four shape-shifting quadrangles* (13·39 = 1 mod 253); *P* = 40, DL = 19 is *five shape-shifting quadrangles* (19·40 = 1 mod 253); and *P* = 38, DL = 20 is *seven shape-shifting quadrangles* (20·38 = 1 mod 253). Additionally, *P* = 41, DL = 37 is *eight shape-shifting pentagons* (37·41 = -1 mod 253) in just under 7 single steps. But *P* = 34, 35, 37, and 43 appear randomly drawn; of course, they are not. All of the static images appear quite similar, but how they are created line by line varies greatly.

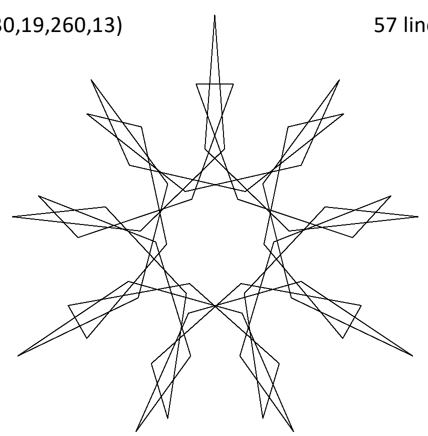
By contrast, the boxed portion of the table, C, shows that the first 7 single-step lengths ***L*** from 7 to 31 span a large number of ***P*** values (it also shows single-step ***L*** value for the top row images on either side of ***P*** = 163). It is unsurprising then that the images do not morph smoothly, as can be seen using *Sequence Player* mode which cycles through 180 total/90 distinct 570-line images. To focus attention on the "jerkiness" of the resulting image sequence, it is worthwhile to slow the speed of the image sequence by changing 100 (default speed) to 200. Compare this with the smoothness of the 220 total/110 distinct 253-line images.

It is interesting to note that the two closest ***P*** value images from part C of the table, ***P*** = 259 and ***P*** = 263, look completely different as static images, despite being separated by only 4 subdivisions. These are at left and right with first single-steps of ***L*** = 11 and ***L*** = 13 shown in red. Between is the 57 -line, SCF = 10 image given ***P*** = 260 and by looking carefully at this image, you can see aspects of the left and right image since there are sharp points like the left image and a spinning star like the cracked-open 13,2 right image.

(30,19,259,13)



(30,19,260,13)



57 lines (30,19,263,13)