Using a Zig-Zag VF to examine Curves and Bundles

As ***n*** varies from 48, the Ultimate Zig-Zag Image sequence produces interesting image sequences when VCF > 1 because the number of lines in the image is reduced from 529***n***, Lines = 529***n***/VCF (***S*** = ***k*** = 23 and 529 = 23^2 for this image sequence). Kicking the Tires discusses some variations on this sequence. Here we focus on small even ***n*** values with VCF = ***n*** or ***n***/2 where the resulting VF is a zig-zag to see open versus closed curves as well as bundles and zero-jump images.

Zig-Zag VF. Given the ***k*** = 23 jump set shown in the second column of Table 1, there are 6 values of ***n*** that produce a zig-zag VF. If ***n*** = 3 a single angle results, and ***n*** = 4 only allows closed vertex curves and a single pair of parallel lines for creating bundles. Vertex jump patterns for the other four ***n*** producing a zig-zag VF are shown as the last four columns of Table 1. Zero jumps, introduced in E17.4.1, are highlighted in yellow. Note that the jump patterns are symmetric about these values, as is required for the VF to fold backwards from ***n***/2 upon itself to create the zig-zag. Two ***n***, 6 and 12, have VCF = ***n***; the other two have VCF = ***n***/2 and hence have 180° rotational symmetry.

n = 6 provides an interesting case study that highlights zero jumps as well as how curves and bundles adjust as ***P*** varies. Table 2 delineates the types of curves and bundles that occur when ***P*** is in the range of various multiples of 23. The best way to understand Table 2 is to play the sequence multiple times slowed down a bit so you can see the transitions.

Table 2. Curves and Bundles

Location of Max ***P*** line				Curves: Closed		Curves: Open		Bundles: Slanted			Bundles: H	Image at Max ***P***
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13 (using Hexagon vertices 0-5)**
1	23	1		4	E							Zig-zag
2	46	5	Z	4	E							Hexagon
3	69	2	Z			2	E	1				0-2-3-5 box + 5-1-4 angle
4	92	4	Z			2	E	2	1			6,2-star plus line from 1-2
5	115	3	Z					1	1	1	1	1-2 and 1-3 bow-ties
6	138	3	Z							3	3	3-diameters + lines 1-3 and 4-5
7	161	4	Z						1	2	2	0-4-5-5-3-0-4-2-1-3 outline
8	184	2	Z			2	R	1	2			6,2-star plus lines 1-4 and 2-3
9	207	5	Z			4	R	1	1			0-5-3-4-1-1-2-3-2-0--5-4 outline
10	230	1	Z	4	R	2	R					Hexagon + 1-5-2-4
11	253	0	Z	8	R							Zig-zag, pausing at each vertex
12	276	0	Z	8	E,R							Zig-zag, pausing at each vertex

Column labels (images in columns 4-12 for *P* in range 23(*m* -1) < *P* < 23*m*).

1 ***m***, multiple of ***S*** = 23.
2 Max ***P***, Maximum ***P*** = 23***m***. ***P*** = 264, 265 = 529/2±0.5 is porcupine.
3 Vertex location of First Line given Max ***P***.
4 Z if a spray of lines from vertex 0 and 3 is present for ***P*** in this range.

***Closed Curves* have intersections at hexagon vertices.**

5 number of curves.
6 E - expanding from vertex, R - receding toward vertex, as ***P*** increases.

Open Curves have intersections outside the hexagon.

7 number of curves.
8 E - expanding from vertex, R - receding toward vertex, as ***P*** increases.

Four types of bundles based on pairs of parallel VF lines.

9 number of bundles using top slanted lines 0-1 and 5-2.
10 number of bundles using bottom slanted lines 3-4 and 2-5.
11 number of bundles using wide slanted lines 0-1 and 3-4.
12 number of bundles using middle horizontal lines 1-5 and 4-2.
13 Image at Max ***P*** (where SCF = 23).

Multiples *m* > 23/2 mirror 23-*m* but reverse direction of curve E and R.

Table 1. Zig-Zag VF Jump Pattern

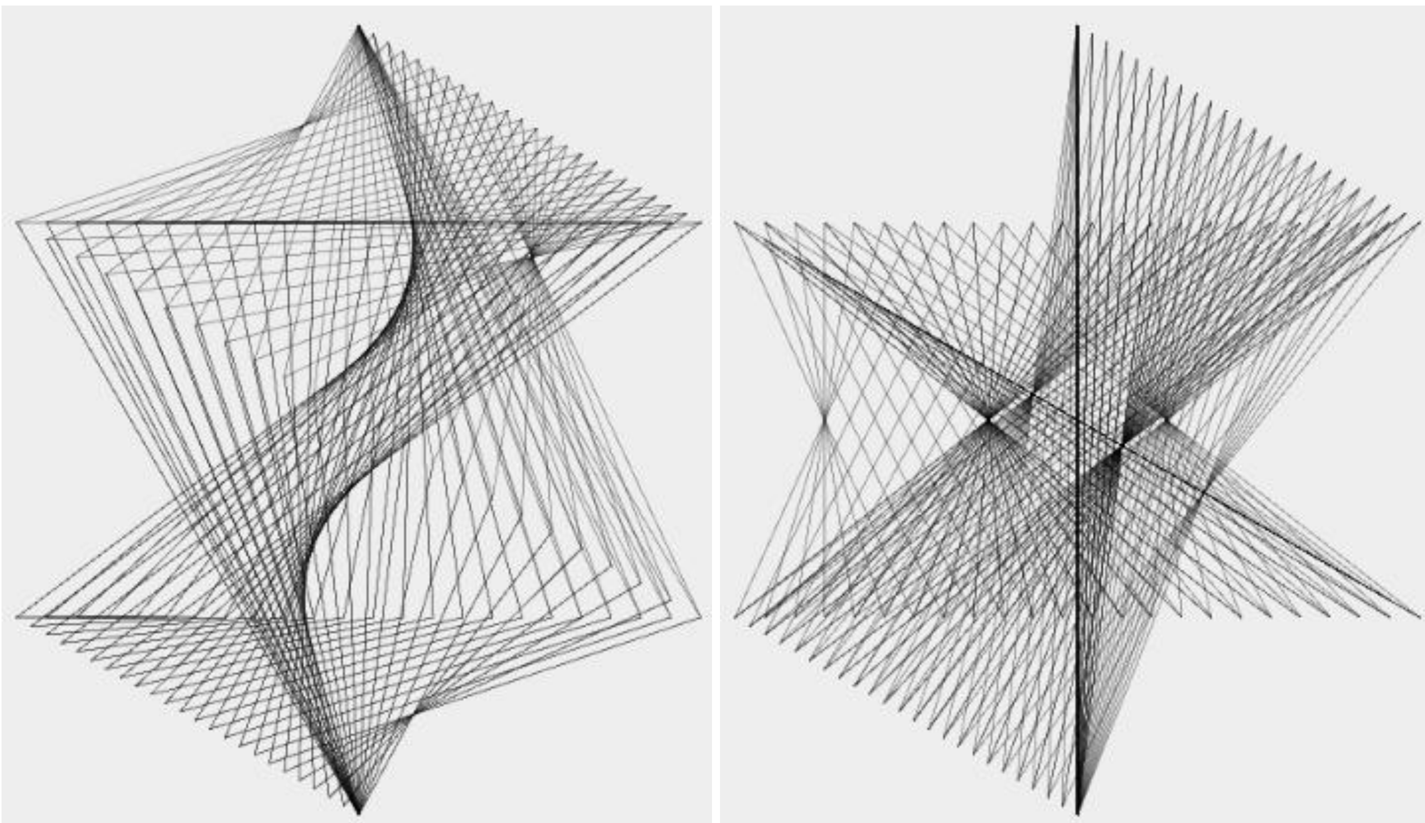
Jump		Sum	^Vertex ***i*** given ***n*** =				
i	***Ji***	Σ ***Ji***	48	6	8	12	24
1	1	1	1	1	1	1	1
2	46	47	47	5	7	11	23
3	3	50	2	2	2	2	2
4	44	94	46	4	6	10	22
5	5	99	3	3	3	3	3
6	42	141	45	3	5	9	21
7	7	148	4	4	4	4	4
8	40	188	44	2	4	8	20
9	9	197	5	5	5	5	5
10	38	235	43	1	3	7	19
11	11	246	6	0	6	6	6
12	36	282	42	0	2	6	18
13	13	295	7	1	7	7	7
14	34	329	41	5	1	5	17
15	15	344	8	2	0	8	8
16	32	376	40	4	0	4	16
17	17	393	9	3	1	9	9
18	30	423	39	3	7	3	15
19	19	442	10	4	2	10	10
20	28	470	38	2	6	2	14
21	21	491	11	5	3	11	11
22	26	517	37	1	5	1	13
23	23	540	12	0	4	0	12

^Vertex ***i*** = MOD(Σ ***ji***, ***n***).

You can also stop the sequence and scroll through ***P*** using *Start Point* until you find a Max ***P*** value then manually move ***P*** to see the image develop over the next (or previous) 22 lines which all have SCF = 1 if you start at a Max ***P*** value.

Porcupines. It is also worth manually examining the porcupine value of ***P*** = 264, 265 here. In the previous section it was asserted that bundles occur close to the porcupine value but in the current context, bundles occur near porcupine/2 values according to Table 2. This is because the 23 jump set produces two zig-zag curves superimposed on one another for ***n*** = 6 as seen in Table 1. (By contrast, bundles are at near porcupine ***P*** for ***n*** = 12; for example, ***P*** = 260 had 9 bundles.)

Static Images. Next, we focus on two images: ***P*** = 75, at left; and ***P*** = 132, at right. The left has two slanted bundles at the top and one at the bottom. The top open curve uses VF lines 0-1 and 2-4 while the bottom uses 3-4 and 5-1. The right has no curves, but 3 slanted wide bundles based on parallel lines 0-1 and 3-4 which have concurrent points on the 2-5 diameter while the 3 middle bundles are based on parallel lines 1-5 and 4-2 and have concurrent points on the horizontal diameter from “vertices” 1.5-4.5 (or from 3 to 9 o’clock on a clock face).



Varying *n*. The final four images show ***P*** = 65 for ***n*** = 6, 8, 12, and 24. Each has a corkscrew shaped center created by ***n***-4 opposing stepped-down open curves. But note that, as we saw in Table 1, the first and third images have VCF = ***n*** so they are rotationally asymmetric (and have a bundle only at the top using parallel lines 0-1 and 2 to ***n***-1). In contrast, the second and fourth have VCF = ***n***/2 so they have a bundle at the top and the bottom due to the 180° rotational symmetry inherent in these images.