On the Constraining Corners of Interior Apexes

An interior apex is created from 4 vertices of the n-gon. These vertices are endpoints for the two lines creating the angle ***a***. You can think of those vertices relative each other with one direction connecting top left, **TL**, to bottom right, **BR**, and the other connecting **TR** to **BL**.

Define. A vertex is a *constraining corner* if it is the corner of the largest (inverted) triangle above or largest triangle below used in an apex count. Corners that are not constraining are *slack*.

An example: Consider the 0-7 and 1-12 intersection annotated to the right (two versions are shown, ***n*** = 22 and ***n*** = 25 with identical right side counts except for one internal apex). **TR** and **BR** are constraining, **TL** and **BL** are slack because the top base line includes vertex 1 (**TR**) of line 1-19 and the bottom base line includes vertex 7 (**BR**) of line 7-13.

Apex counts are based on the lines in the base line direction. The apex count for this point is therefore **7**.

We next consider what happens along one of the two lines as we move lower in the image.

Along **TL-BR**. **BR** is still constraining but the parallel line one to the right of TR-BL (2-11) is now constraining at the new TR location of vertex 2. Total apex count is therefore one smaller, or **6** in the example image. This continues for the remaining interior apexes along this line (or **7**, **6**, **5**, **4**, **3**) in both images. Note that the same pattern of decline occurs on other lines in the TL-BR direction.

Along **TR-BL**. **TR** is still constraining but the parallel line one to the left of **TL-BR** (21-8 top, 24-8 bottom) is now constraining at the new BR location, vertex 8, AND at **BL** or vertex 12. Total apex count is therefore one larger, or **8** in the example image. This same count occurs for additional internal apexes on **TR-BL** until **TR** is no longer viable as the largest inverted triangle base.

Compare the last apex intersecting with **TL'-BR'**. **TL'** is slack in the bottom but constraining in the top image. The largest inverted triangle uses base line image **TL'-TR+1** at top (18-2) but remains the one through **TR** at bottom (1-19). Therefore, top image apex counts are **7**, **8**, **8**, **8**, **8**, but **7**, **8**, **8**, **8**, **8** at the bottom).

Other buildup patterns. The internal buildup of apex counts on each arc increases by 2 because both endpoints on a side are constraining factors. This applies to both L and R. This increase continues until ***b***+1 is achieved (8). There are ***a***-1 arcs (5) and they vary between starting at internal apex counts of 2 and 3 and need not be symmetric as can be seen at bottom left.

Hint: The easiest way to see the overall pattern is to work from the sides, arc by arc until you reach the plateau value of ***b***+1. The arcs start between the smallest vertex apex counts on each side (vertices 3 and 7 on the right, and between 14 and 18 at top and 15 and 20 at bottom left).