Analyzing Edge and Interior Apex Arc Count Patterns for Scalene Triangles Images

The 21 ***J*** = ***V*** = ***b***, ***K*** = ***W*** = ***a***+***b*** vertex triangle configuration partial images on the next page provide a systematic view of how internal apex arcs start. The first two rows vary ***n*** = 3***b*** for ***b*** = 3 to 9 and the third row varies ***a*** and ***c*** while maintaining ***a***+***c*** =20 holding ***b*** =10 so that ***n*** = 30. These values were chosen because in each case, ***b***, in the bottom right corner of the vertex triangle is 1/3 of the way around the 3***b***-gon (at (***x***, ***y***) = ($3^{0.5}$/2, 0.5)). The patterns described below are based on these 21 images, each of which has a vertex triangle at 0-***b***-***a***+***b***.

For both even and odd ***a***, there are ***a***-1 internal arcs. The internal arc starting values alternate between **2** and **3**.

When ***b*** is even, the smallest segment in the ***b*** direction is 2 (like 3-5 in top row, second from right). In this event,

- The ***b***/2 vertex has 2 lines and apex count of 0 (0s omitted). The smallest perimeter apex count is **2** at vertex ***b***/2-1.
- The top internal arc starts with **2** just below vertex ***b***/2 with bases emanating from adjacent vertices ***b***/2 and ***b***/2+1.
- Since vertex apex counts decline by **2** clockwise from vertex 0, the smallest apex count is **2** at ***b***/2-1.

When ***b*** is odd, the smallest segment in the ***b*** direction is 1 (like 4-5 in top row, right). In this event,

- It spans (***b***-1)/2 to (***b***+1)/2. The smallest perimeter apex count is **1** at (***b***-1)/2.
- If ***a*** > 1, the first internal arc starts just below (***b***-1)/2 and has value **3** with top and bottom bases separated by two vertices with top base at (***b***-1)/2 and bottom base at (***b***+1)/2+1.
- If ***a*** > 2, the second internal arc starts just below (***b***+1)/2 and has value **2** with bases at (***b***+1)/2 and (***b***+1)/2+1.

We can find the start of inverted perimeter apex counts by focusing on the parallel lines in the ***a***+***b*** direction.

- By construction, the first two rows had first inverted apex at vertex ***b***.
  - In Row 1 that count is **1** in Row 2 that count is **2**.
- More generally in Row 3, when ***a***+***b*** is odd, the first inverted perimeter apex count is **1** at vertex (***a***+***b***+1)/2.
  - However, when ***a***+***b*** is even, the first inverted perimeter apex count is **2** at vertex (***a***+***b***)/2+1.

Even ***n*** images have rotational symmetry so that the patterns noted are rotated 180° on the left hand side.

Odd ***n*** images have a different edge pattern because if the smallest segment is **1** on left it has a **2** on right and vice versa.

The three images at right. These images offer a counterpoint to the 21 on the next page. Specifically, they match the ***a***, ***b***, ***c*** values in the right-most column using the no-VT configuration.

Each uses the ***J*** = ***b+1***, ***K*** = ***W*** = ***a+b+1***, and ***V*** = ***b*** configuration discussed elsewhere. The first two are odd ***n*** (***n*** = 27) and hence have a vertex triangle that starts just past the bottom at (***n***+1)/2 (14-23-4 for the first image and 14-23-3 for the second image).

The third image has the same ***a*** = 7, ***b*** = 10 and ***c*** = 13 values as the bottom right image on the next page. This version has no vertex triangle, and the perimeter and internal apex patterns are the "reverse" of one another.

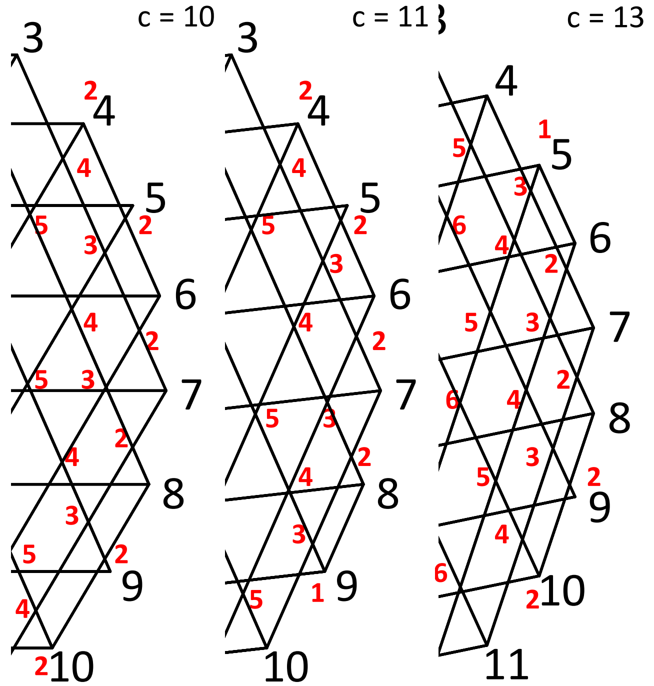
Clockwise perimeter end: Last count of **1** at vertex 5 vs. **2** at 4.

Six interior arc starts of **3**, **2**, **3**, **2**, **3**, **2** vs. **2**, **3**, **2**, **3**, **2**, **3**.

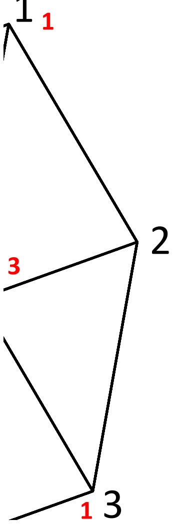
Clockwise perimeter start: Count of **2** at vertex 10 vs. **1** at 9.

An isosceles caveat. With ***b*** = ***c*** interior apex counts may change due to concurrency along diameters.

a = 8 b = 9 c = 10 | a = 7 b = 9 c = 11 | a = 7 b = 10 c = 13



a = 2
b = 3
c = 4



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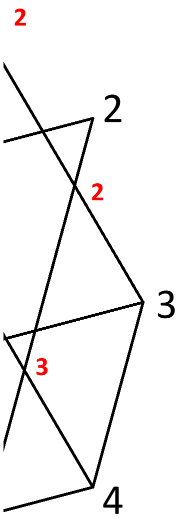
c = 5



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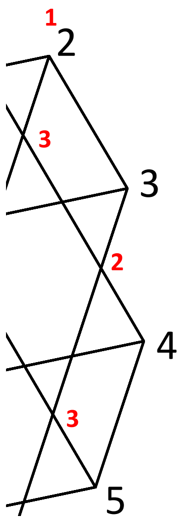
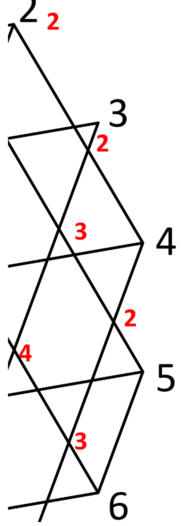
c = 6



a = 5
b = 6
c = 7



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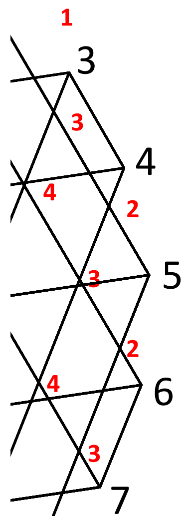
c = 8



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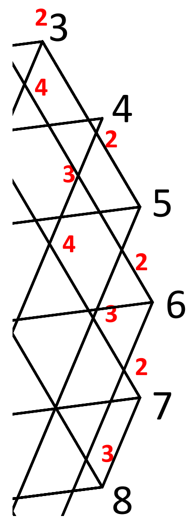

a = 8
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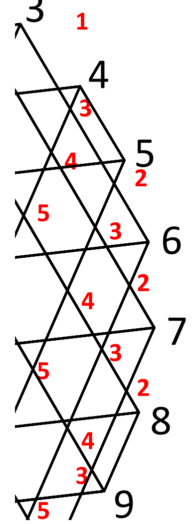
c = 10



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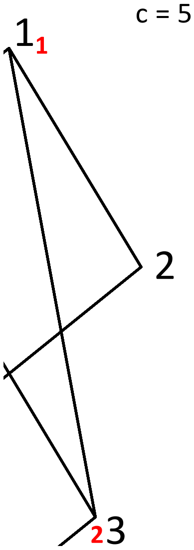
c = 5



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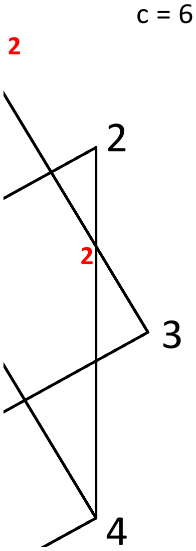
c = 6



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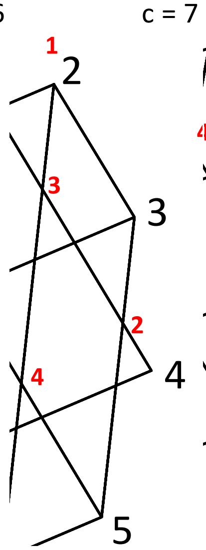

a = 4
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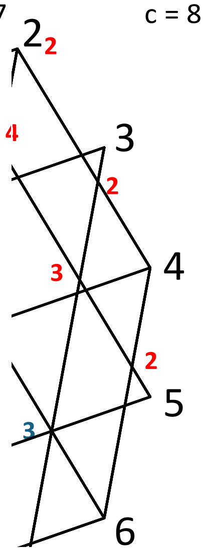
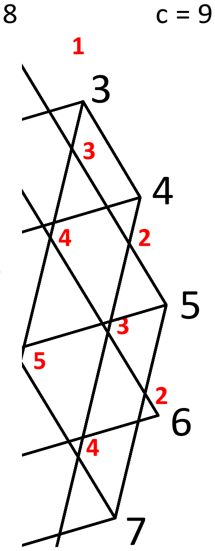
c = 8



a = 5
b = 7
c = 9



a = 6
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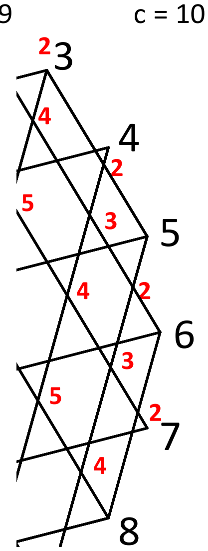
c = 10



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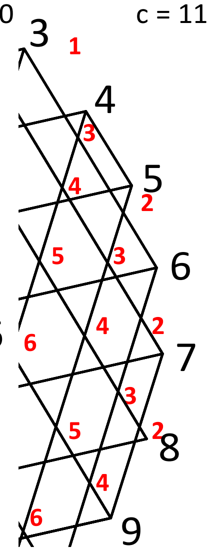

a = 1
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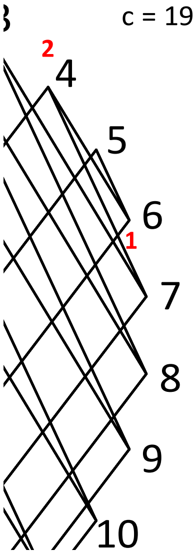

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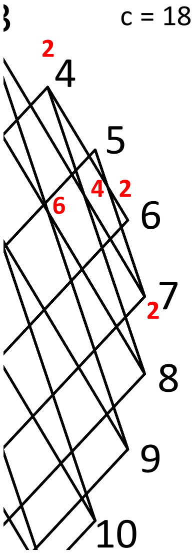

a = 3
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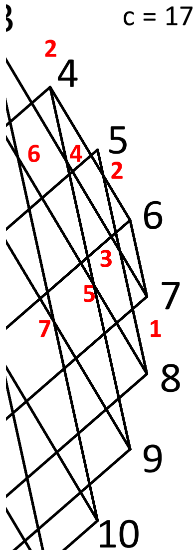

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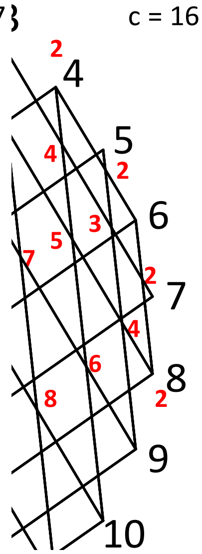

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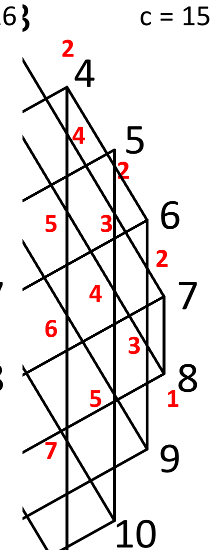
c = 15



a = 6
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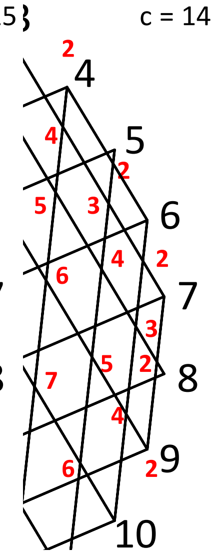

a = 7
b = 10
c = 13