Rotated, Reflected, and Standard Form Images

When you are initially playing with the *General Triangles* file it is often worthwhile to simply input numbers in for ***JKVW*** and see what happens. As you dig deeper into these images you may find it useful to input equations for the various parameters, such as the example of creating isosceles right triangles images discussed in Section 2.5.

Any set of ***JKVW*** values that produce 3 distinct directions will create a triangles image. This image can be described by the three angles in each triangle, ***a***, ***b***, and ***c***, with ***a*** ≤ ***b*** ≤ ***c*** with ***a***+***b***+***c*** = ***n***, derived from ***JKVW*** and ***n*** (as explained in blue shaded cells M6:X9 of the *General Triangles* file) and by whether the image has 0, 1, or 2 vertex triangles (VT). We know from the previous section that if ***n*** is odd, there is one VT and if ***n*** is even there are either 0 or 2. The odd ***n*** version has one because there is not rotational symmetry, and the even ***n*** version has either zero or two due to ***n***/2 rotational symmetry (so that flipping the image 180° produces the same image). Both standard forms key off angles ***a*** and ***b*** which makes them useful if you want to explore what happens to an ***a***,***b***,***n***-***a***-***b*** image as ***n*** varies so long as ***n*** ≥ 2***b***+***a***. When ***n*** = 2***b***+***a***, the image is isosceles; for larger ***n***, the images is scalene (unless ***a*** = ***b***).

Standard Form VT. Set ***J*** = ***V*** = ***b*** and ***K*** = ***W*** = ***a***+***b***. This produces an image with a vertex triangle at 0–***J***–***K***–0 where the – signs mean the line between vertices. If ***n*** is even there is a second VT at ***n***/2–(***n***/2+***b***)–(***n***/2+***a+b***)–***n***/2 where the third vertex is mod ***n*** because it is certainly possible that ***a***+***b*** ≥ ***n***/2. (It is simpler to call the vertex triangle 0***JK*** but to describe the vertex triangle starting at the bottom it makes sense to be more explicit here.)

Standard Form no VT. Set ***J*** = ***b***+1, ***V*** = ***b*** and ***K*** = ***W*** = ***a***+***b***+1. This produces an image with angle ***a*** at 0 since ***J***0***K*** spans ***a*** vertices, but there is no vertex triangle starting at 0 (the side opposite angle ***a*** is ***VW*** which intersects 0***J*** on the interior of the ***n***-gon. If ***n*** is odd, the apex angle of the VT in the no VT standard form starts just to the left of the bottom at vertex (***n***+1)/2.

Both standard forms create images where the ***a*** angle ***J***0***K*** opens towards the right (unless ***b*** = ***c*** in which case it is an isosceles triangles image symmetric about the vertical diameter). The downward pointing ***a*** angles open more and more to the right as ***n*** increases. The image you obtain using the standard form may well not look like the image you have via different versions of ***JKVW***, but your image can be obtained from the standard form image by rotation and reflection.

Rotating an Image. From any starting point, you can rotate an image clockwise 1/***n*** (one vertex) by increasing ***J***, ***K***, and ***W*** by 2 for fixed ***V***. To rotate an image counterclockwise, simply subtract 2 from these same three values while keeping ***V*** fixed.

To obtain roughly vertical apex angles. One use of this strategy is to rotate an image so that the apex angle, ***a***, is roughly vertical to make counting easier. Starting from the standard form, round (***c***-***b***)/4 to the nearest whole number then multiply by 2. Add this to ***J***, ***K***, and ***W*** for fixed ***V***. This works by moving the apex angle at 0 to roughly surround ***n***/2.

Reflecting an Image. As noted above, the standard form tilts the apex angle at 0 to the right. If you want it tilted to the left, then simply replace ***JKVW*** with ***n*** minus these values, or, to create the same result, replace ***J*** with -***J***, ***K*** with -***K***, **V** with -***V*** and ***W*** with -***W***. Both methods produce the same result due to the modular nature of vertices of the ***n***-gon.

Simplifying *JKVW* Data Entry. If you enter ***JKVW*** values in the unprotected green cells in the *General Triangles* file in the order shown and link the unprotected yellow cells in Q4:W4 to these cells, you can very easily adjust between VT and no VT for fixed ***a***,***b***,***c*** as well as adjust across images. It helps to follow through this example using the *General Triangles* file.

6	***V***
6	***J***
9	***K,W***

The letters are out of order for a reason: ***V*** is the same for standard form VT and no VT. The values shown are associated with a standard form 3,6,***n***-9 VT image. Change to 3,6,***n***-9 no VT by increasing the last two numbers by 1 to 7 for ***J*** and 10 for ***K*** and ***W.*** Next, increase ***V*** by 1 to examine 3,7,***n***-10 VT images and repeat. This was the strategy used to systematically cycle through the 422 images with ***a*** > 1 for ***n*** < 25 to find images with interior points of concurrence as discussed in Section 7.3.