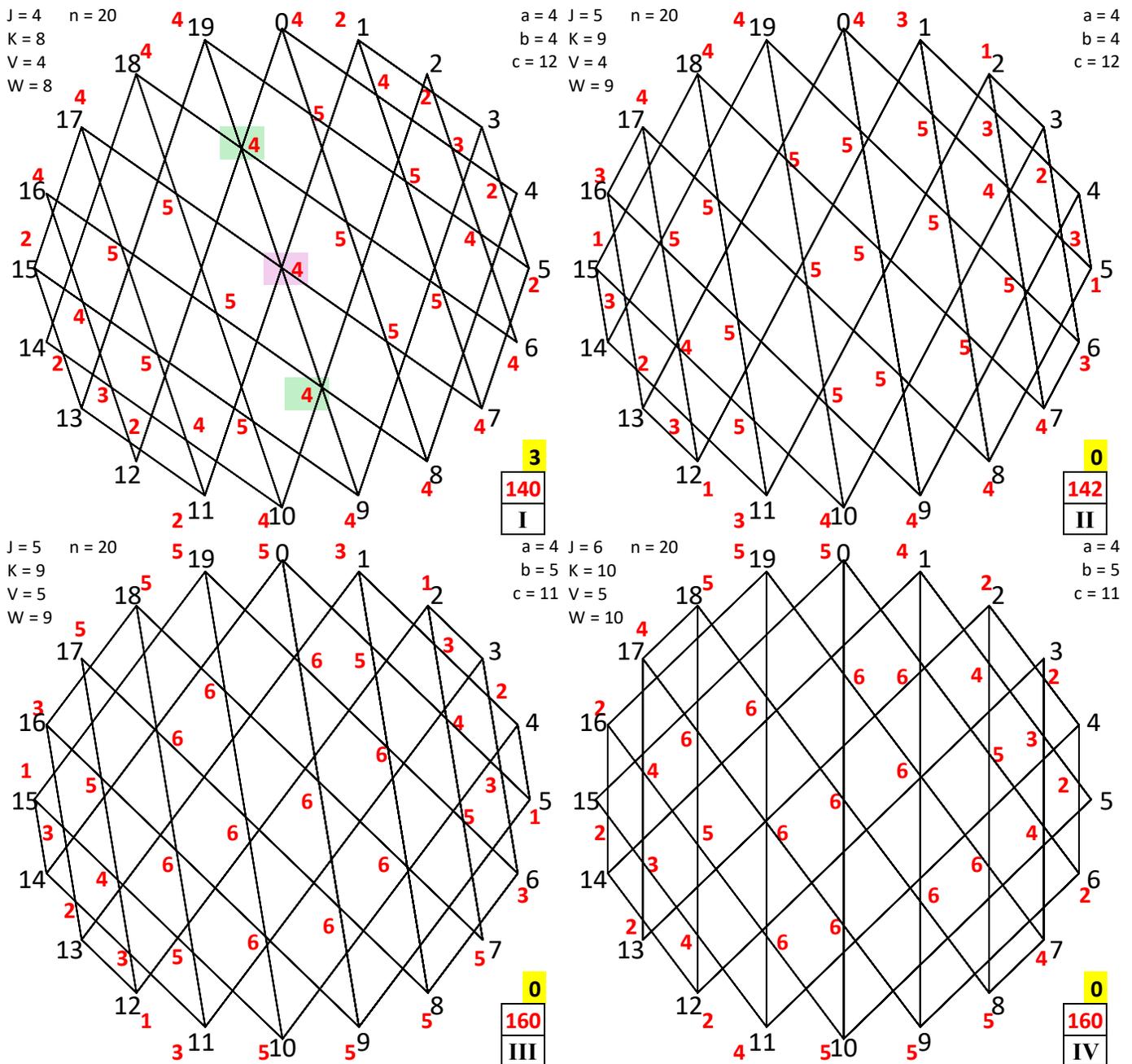


An Introduction to Concurrency

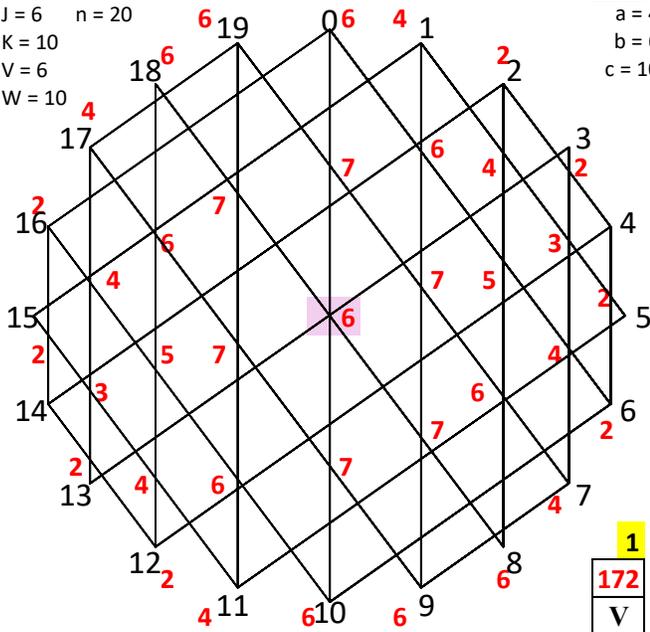
The discussion earlier in this chapter, but especially in the previous section, provides a roadmap for answering how many triangles are in a triangles image. We see that the answer is more complex if n and a are even. As a result, we focus attention on $n = 20$ and $a = 4$ because these values are small enough that the patterns that emerge are readily visible without being excessively complex.

Ten images are shown, two each for $b = 4$ through 8, the VT version is on the left and the no VT version is on the right. At bottom right of each image are three elements: the number of concurrences highlighted in yellow; the total triangles count in red; and image label, I-X, are boxed. Note that B and C from the previous section are images V and VI.

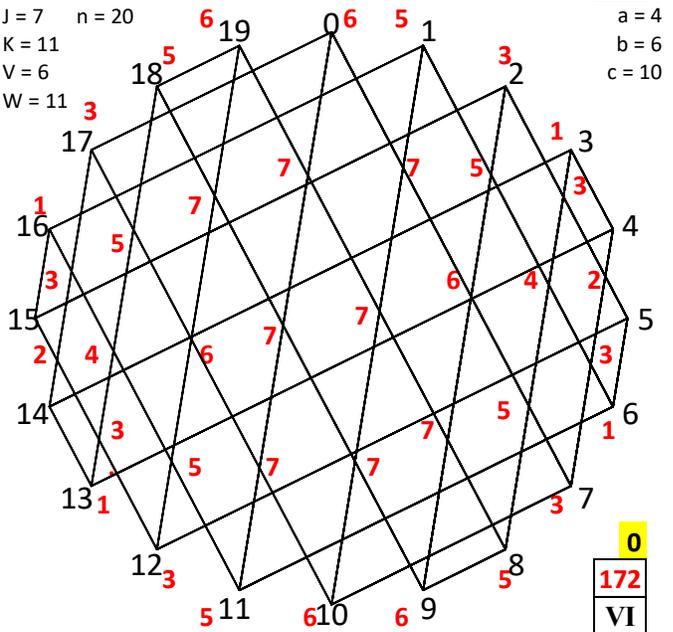
The goal here is not to formally discuss the formulas that characterize these images, but to point out why those formulas lead to the results seen. The actual results depend on the presence of points where three lines intersect at a single point on the interior of the image. These are points of concurrence and each one reduces the number of triangles in the image by one. There are two forms of systematic concurrence and one that is more sporadic. We see examples of each type of concurrence in the images that follow. The next chapter provides a more expansive discussion of this issue.



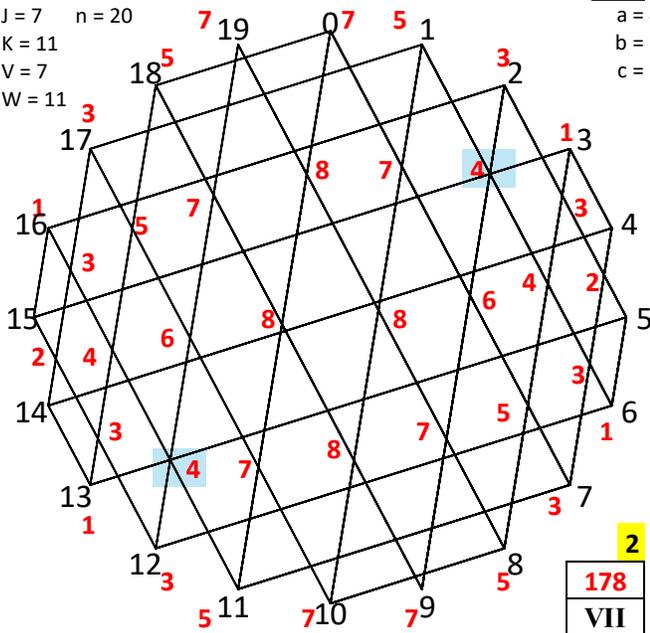
J = 6 n = 20
 K = 10
 V = 6
 W = 10



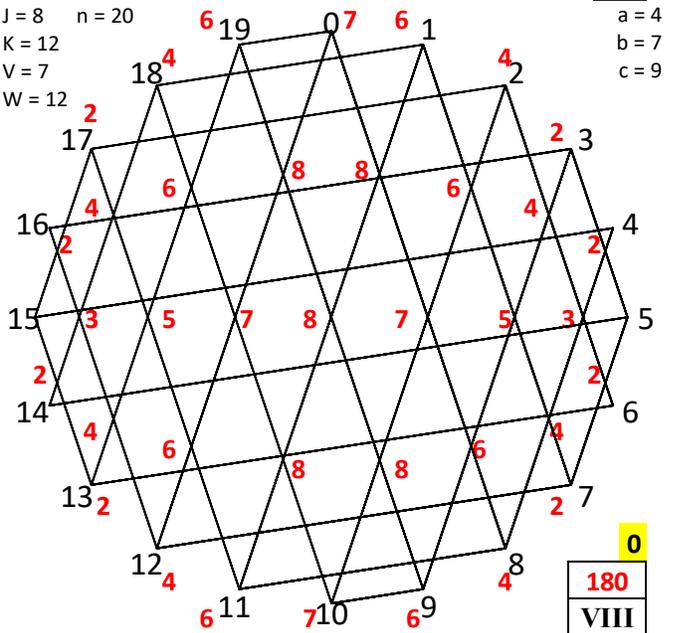
a = 4 J = 7 n = 20
 b = 6 K = 11
 c = 10 V = 6
 W = 11



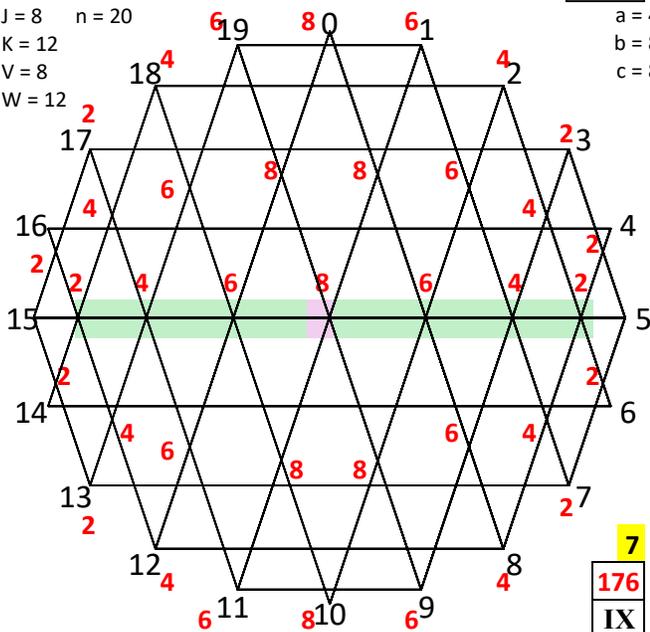
J = 7 n = 20
 K = 11
 V = 7
 W = 11



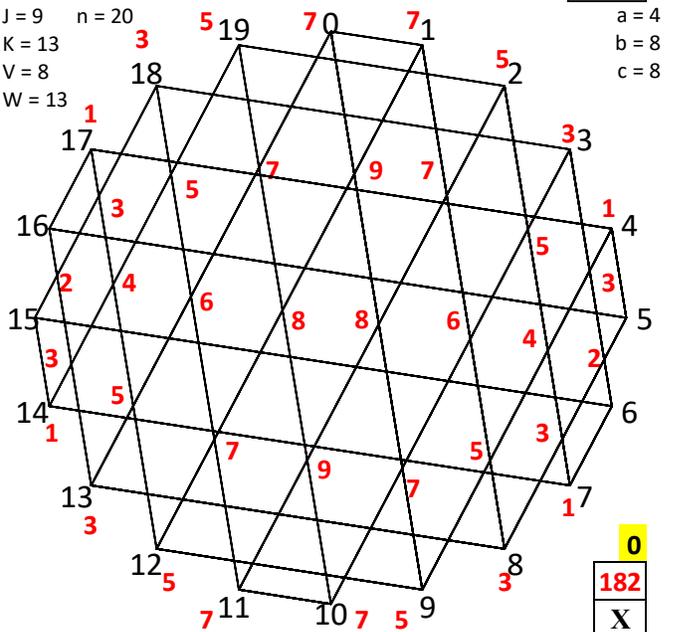
a = 4 J = 8 n = 20
 b = 7 K = 12
 c = 9 V = 7
 W = 12



J = 8 n = 20
 K = 12
 V = 8
 W = 12



a = 4 J = 9 n = 20
 b = 8 K = 13
 c = 8 V = 8
 W = 13



a = 4
 b = 6
 c = 10

a = 4
 b = 7
 c = 9

a = 4
 b = 8
 c = 8

To see patterns, ignore concurrence. Six of the ten images do not have internal points of concurrence but four do have such points. If you examine the five pairs of triangles totals noted in **red** in each image and summarized in the table, it is difficult to discern a pattern. The problem has to do with internal points of concurrence. A point of concurrence interrupts the internal apex count pattern.

The easiest interruption to see is the one in VII in the upper and lower arcs. Since $b = 7$ in this image and since the upper and lower arcs start with **3** (since the smallest perimeter triangles count is **1**), the pattern *should* increase by 2s starting at **3** to **7** then peak at **8** ($8 = b+1$). That pattern is interrupted by the **4** between **3** and **7** due to the point of concurrence on each arc.

If you add the points of concurrence to the total triangles count a pattern becomes apparent. The Pattern rows in the table show that a different result occurs if b is even from when b is odd. In particular, the VT and no VT Pattern counts are the same if b is odd, but VT is one larger than no VT if b is even.

Three types of concurrence. Four of the ten images have points of concurrence and they are highlighted in different colors. The easiest to conceptualize is a central point of concurrence, highlighted in plum, since it only occurs at the center of the polygon. The three lines which coincide in this instance are diameter lines. Due to the symmetry inherent in even n images, all other points of concurrence come in pairs.

A second form of concurrence, highlighted in green, includes a diameter line and two other lines. These lines are created in isosceles triangles images like VT images I and IX. However, not all isosceles triangles images have diameter points of concurrence as we see by noting that the no VT images II and X have no diameter lines.

The first two forms of concurrence have well-defined rules for when they will occur, as we will discover in the next chapter. The final form, sporadic concurrence, highlighted in blue, is not so easily codified. We see an example of sporadic concurrence in image VII.

A final note. One might conclude from the ten images that no VT style images have at least as many or more triangles than VT style images. In general this is not true, but has to do with n being divisible by 4. Had we chosen an n divisible by 2 but not 4, like 18 or 22, the reverse could hold. Consider three examples from $n = 22$. The VT version of (4,6,12) has 206 triangles but the no VT version has 204 due to a central concurrence in the no VT version. The VT version of (4,9,9) has 228 triangles but the no VT version has 220 due to 8 diameter concurrences in the no VT version. And the VT version of (5,7,10) has 242 triangles but the no VT version has 240 due to 2 sporadic concurrences. As we see in the next chapter, none of the VT images for $n = 22$ have concurrences just like none of the no VT images for $n = 20$ have concurrences, not just the five no VT images shown above.

| Total Counts | Even b | | Odd b | |
|--|------------|------------|------------|------------|
| | I | II | III | IV |
| Image | | | | |
| Top | 24 | 24 | 28 | 27 |
| Inside | 92 | 94 | 104 | 106 |
| Triangles | 140 | 142 | 160 | 160 |
| Concurrences | 3 | 0 | 0 | 0 |
| Pattern | 143 | 142 | 160 | 160 |
| Image | V | VI | VII | VIII |
| Top | 30 | 30 | 32 | 31 |
| Inside | 112 | 112 | 114 | 118 |
| Triangles | 172 | 172 | 178 | 180 |
| Concurrences | 1 | 0 | 2 | 0 |
| Pattern | 173 | 172 | 180 | 180 |
| Image | IX | X | | |
| Top | 32 | 32 | | |
| Inside | 112 | 118 | | |
| Triangles | 176 | 182 | | |
| Concurrences | 7 | 0 | | |
| Pattern | 183 | 182 | | |
| Triangles = 2·Top+Inside | | | | |
| Pattern = Triangles +Concurrences | | | | |