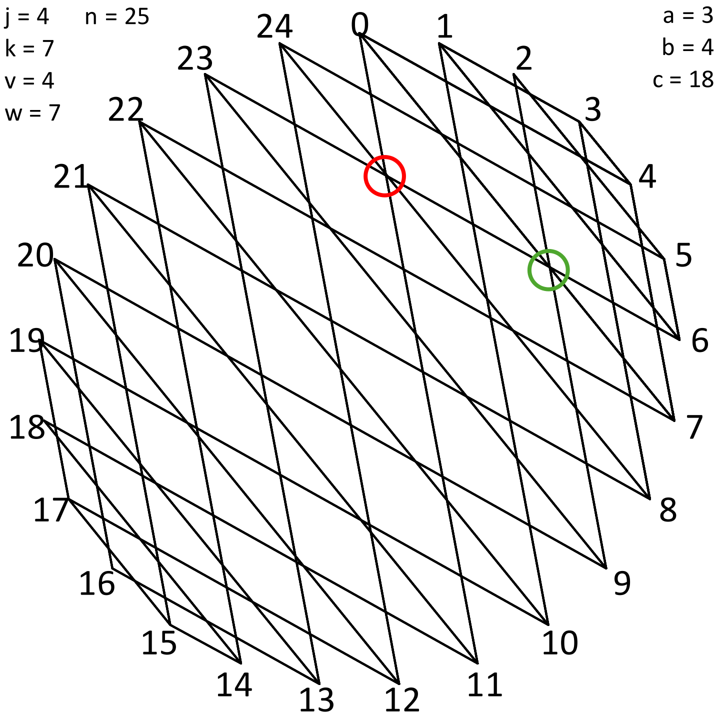
Near Concurrence and Adjusting *n* to Find Smallest Triangles

Often you will find an image that appears to be concurrent that has an odd ***n*** but when you nudge ***n*** in one direction or the other you will find an even ***n*** where the “same” three lines are concurrent. But there are times when this does not occur as well. The *Smallest Triangle* version of the concurrence file allows you to quickly find situations where both situations occur. [If a line is vertical, rotate the image ⟳ 1/***n*** by increasing ***J***,***K*** and ***W*** by 2 which increases ***j1***-***j6*** by 1.]

If the vertices defining the lines are all in the first ¾ or so of ***n***, then you need not adjust the endpoints to changes in ***n***. On the other hand, if some of the vertices are in the last ¼ of ***n***, you should use an equation in the highlighted green cells for those vertices so that the location of the vertices relative to vertex 0 is maintained.

An Example. The 3,4,18, ***n*** = 25 shown below has two apparent concurrences. The one **circled in red** is examined in Table 1, an *Excel* screenshot to the right. This is NOT a point of concurrence, but if you use the equation =n-1 for ***j4*** and =n-2 for ***j6*** you can adjust ***n*** via the up/down arrows and those two vertices will adjust accordingly.

j = 4   n = 25
k = 7
v = 4
w = 7



a = 3
b = 4
c = 18

n	25		
0	j1	Line 1 (*n*-gon vertices)	
11	j2		
8	j3	Line 2 (*n*-gon vertices)	
24	j4		
6	j5	Line 3 (*n*-gon vertices)	
23	j6		

Concurrent Points 3 lines are concurrent when they p

***Smallest Triangle* Locked Cells Version: Numbers or equ**

Testing for one concurrent point on one image. This

Solve for x in mx+b = cx+d (1-2)
x* = (d-b)/(m-c)
0.082321 x*

25 *n*
Concurrent if x* and y* are equal.

Lines 1&2 Check: Same y at x*.
0.56846 mx+b
0.56846 cx+d

Lines 2&3

	π fraction	x	y
0 *j1*	0	6.1E-17	1
11 *j2*	0.8800000	0.36812	-0.92977649
8 *j3*	0.6400000	0.90483	-0.42577929
24 *j4*	1.9200000	-0.2487	0.968583161

Line y = mx+b from j1-j2
Δy/Δx=m -5.24218
y_0-mx_0=b 1

Line y = cx+d from j3-j4
Δy/Δx=c -1.20879
y_0-cx_0=d 0.667969

Line 1 x Line 2 is
Point A

	π fraction	x	y
6 *j5*	0.4800000	0.998027	0.062791
23 *j6*	1.8400000	-0.48175	0.876307
8 *j3*	0.6400000	0.904827	-0.42578
24 *j4*	1.9200000	-0.24869	0.968583

Line
Lin

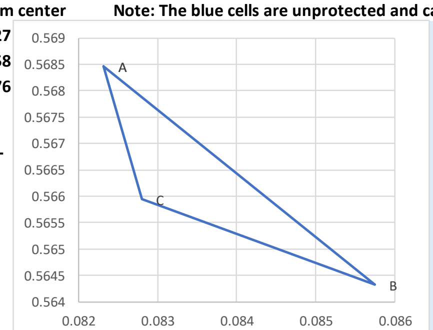
$\pi_{fraction}$ = 2****j***/***n***   x = COS((0.5-$\pi_{fraction}$)*pi())   y = SIN((0.5-$\pi_{fraction}$)*pi())   wh

x value	y value		distance from center
0.0823206	0.56846	A	0.574389827
0.08574	0.56432	B	0.570799158
0.08280	0.56594	C	0.571965076
0.0823206	0.56846	A	

Note: The blue cells are unprotected and ca

Distance between lines intersection points (0 for concurrence):		
		n
		25
		Js
		0
0.00537	A-B	11
0.00336	B-C	8
0.00257	A-C	24
3.319E-06	Δ Area	6
1.2990381	Max Area	23

Table 1



In crème at the bottom of the screenshot you can see the lengths of the three legs of the triangle **ABC**. Also shown there is the area of the resulting triangle (3.3E-06). If you nudge ***n*** smaller to ***n*** = 24, the orientation of the triangle is the same but the size has increased to 4.4E-05. If you nudge ***n*** larger to ***n*** = 26, the orientation of the triangle switches (the longest side is still **A-B**, but **A** is now at the bottom with **B** at the top and **C** above line **AB**) and the size has increased to 4.1E-06 as can be seen in Table 2. These observations show that ***n*** = 25 is closest to concurrent for this set of lines.

x value	y value		distance from center
0.1021686	0.58549	A	0.594333265
0.09812	0.59005	B	0.598154209
0.10148	0.58829	C	0.596979174
0.1021686	0.58549	A	

Note: The blue cells are unprotected and ca

Distance between lines intersection points (0 for concurrence):		
		n
		26
		Js
		0
0.00610	A-B	11
0.00379	B-C	8
0.00289	A-C	25
4.095E-06	Δ Area	6
1.2990381	Max Area	24

Table 2

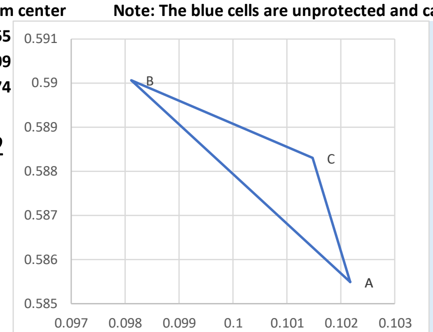


Table 3

Distance between lines intersection points (0 for concurrence):		
		n
		24
		Js
		0
0.00000	A-B	7
0.00000	B-C	2
0.00000	A-C	9
1.778E-16	Δ Area	6
1.2990381	Max Area	22

Following the same strategy for the apparent point of concurrence **circled in green** (by changing ***j2***-***j4*** to 7, 2, and 9) produces the 9.9E-06 sized **ABC** triangle for ***n*** = 25. But here, when we nudge ***n*** down to ***n*** = 24, concurrence occurs as seen in Table 3 (with side lengths 0.00000 for all three lines and the area of the triangle is 1.8E-16 which is due to rounding at 15 digits for trigonometric functions in *Excel*).