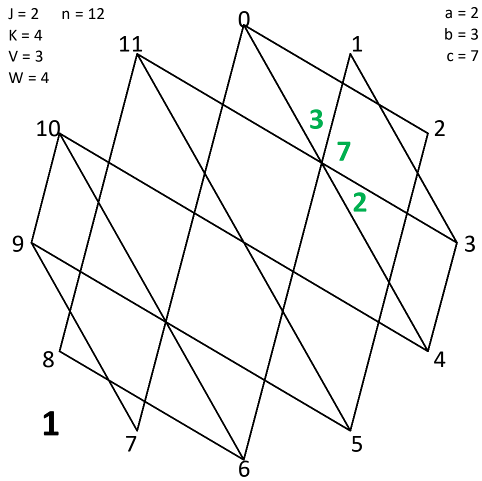
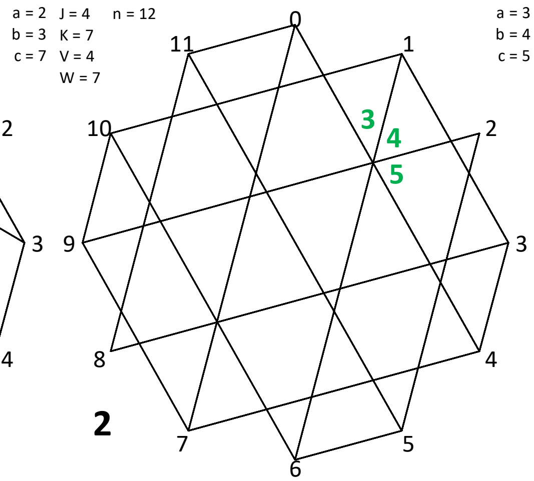
Four Lines Concurrences Create Two Triangles Images with a Common Concurrence Distance

n* = 12.** Triangles images 1 and 2 are the smallest ***n for which two distinct scalene triangles have concurrence points the same distance from the center (at 0.518 of the radius). Image 1 has been rotated one vertex counterclockwise from its standard orientation (Image 3 of Section 7.2) so that the points of concurrence line up, vertex by vertex, and angles are noted in **green** (degree measures for these angles are obtained by multiplying these values by 15 = 180/***n***).





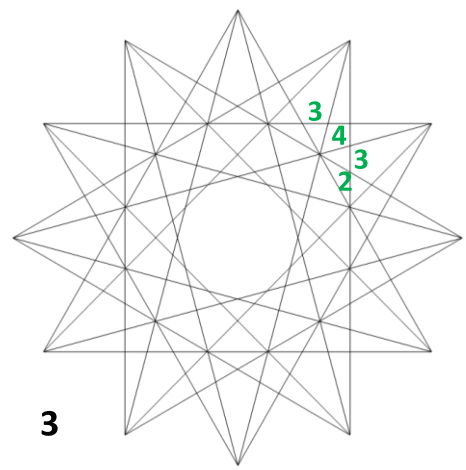
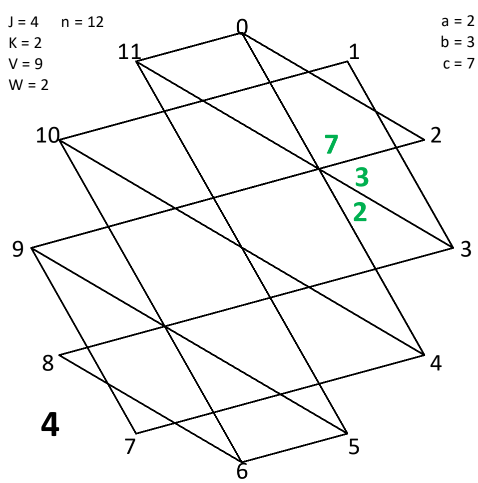


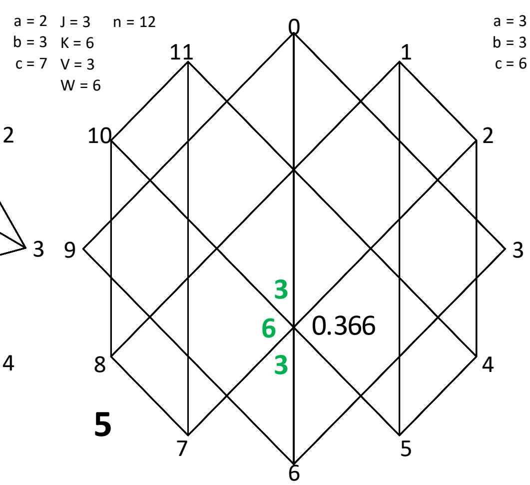
Image 1 has two lines of length 4 and one of length 5 at the concurrence point (2 even and one odd, hence no VT) and image 2 has one line of length 4 and two of length 5 at the concurrence point (1 even and two odd, hence VT).

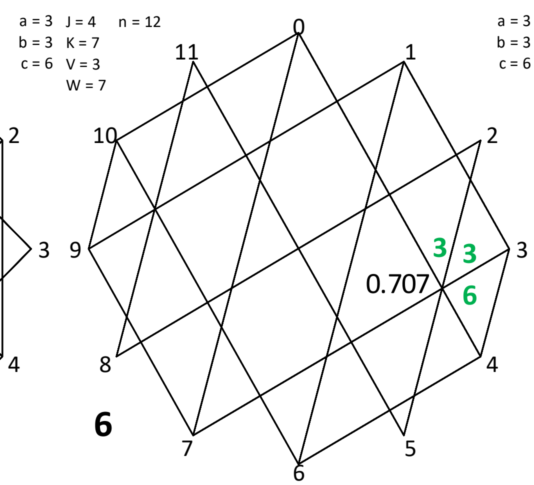
About the concurrence point in the first quadrant. Two of the three lines are the same, 0-4 and 1-6 in both images; the angle created by these lines is **3**. The "missing line" in image 1 is 2-9 and in image 2 it is 3-11. If we manually place a line from 2-9 in image 1 or a line from 3-11 in image 2, we obtain a 4 lines concurrence point at 1.5 on the clockface as discussed in the final part of Section 7.2. The angles thus created are **3**, **4**, **3**, **2** for the half circle above the line 0-4 just like with the stacked 12,4+12,5-star shown in image 3 with **angles** noted.

The order of the numbers matters. Triangles images require three angles. Given four numbers representing angles that sum to ***n*** (each of which must be 2 or more to create an interior concurrence), we can readily see why some combinations and not others produce images which concur at a given point. Adding the 2nd and 3rd angles from image 3 produces **7** and image 1, but adding the 3rd and 4th produces **5** and image 2. Had we added the first and second, we would have ended up with the reflected and rotated version of image 1. This is shown as image 4. This version simply removes the line from 1-6 so that **7** is the first angle seen in the half-circle above 0-4.

This addition must involve adjacent angles. For example, adding the 2nd and 4th angles produces a 3,3,6 set of angles (or isosceles right triangles) and 1st and 3rd produces 2,4,6 images but these images do not have concurrent points at the same distance from the origin. 3,3,6 images 5 and 6 have concurrent distances noted; 2,4,6 VT has a central concurrence.







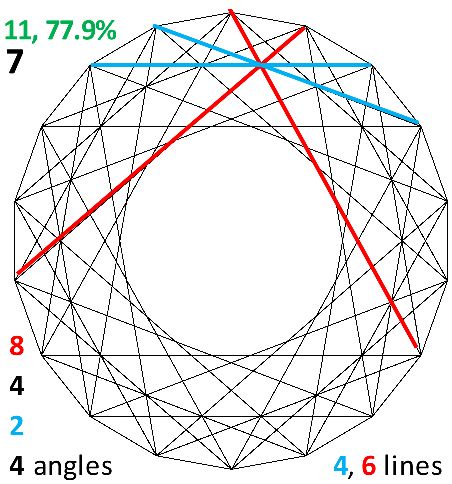
This happens whenever there is a 4 lines concurrence. Two distinct triangles images have concurrences at this distance.

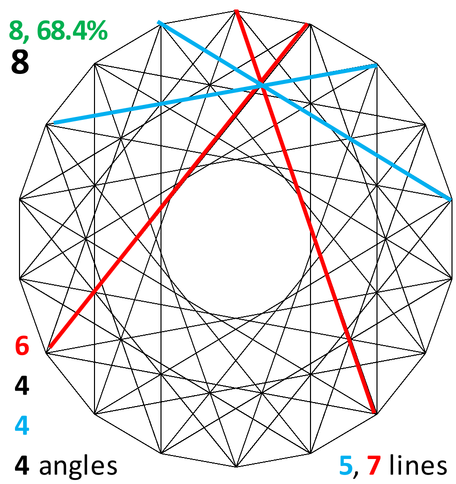
n* = 18.** We saw three more examples of 4 lines concurrence with ***n = 18. Additional examples will arise for larger ***n*** divisible by 6 but it suffices to examine what happens for ***n*** = 18 to see the general pattern that emerges in this instance.

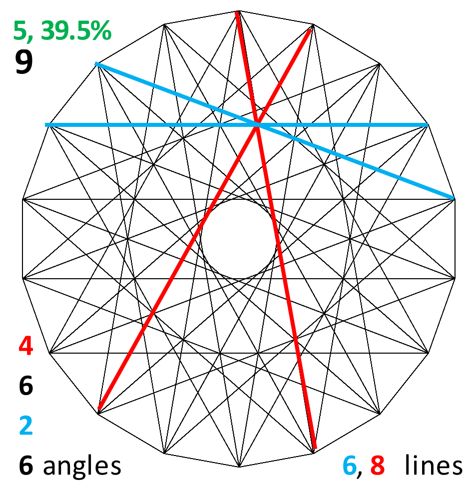
Images 7-9 reproduce Figures 3.4-3.6 from the previous section with some additional information. Each has two pairs of lines intersecting at a single point. The longer of the two pairs is in **red** and the shorter is in **blue**. Four angles are created at the common intersection point: those that are created by lines of the same color, noted in **red** or **blue**, and those that have one line of each color, noted in **black**, at bottom left in each image.

Symmetry. There is symmetry about the diameter through the point of intersection. This means that the angles created by lines spanning different numbers of vertices (created by the **red**/**blue** lines) must be the same. Put another way, the **black** angles must be equal. If we denote the angles by color rather than size, the pattern starting with **red** is **A**, **B**, **C**, **B**. Since opposing angles are equal, it does not matter where we start our analysis: *the angles created by the two different sized lines will be the same size.*

This same pattern exists for image 3 above, the longer lines, spanning 5 vertices, create angle **4** with next two angles being **3**, **2**, **3** and we see that both angles created by lines spanning 4 and 5 vertices are **3**. This is why the first and third angles were the same in image 3 above; in that instance, the two longer lines created angle **4** as we have just noted.







Angle triples: 2,4,12 and 4,6,8 4,4,10 and 4,6,8 2,6,10 and 4,6,8

Angle triples. Given four angles **A**, **B**, **C**, **B** where the sum of these angles is ***n***, we obtain triples by considering what angles we can create by adding two adjacent angles. We obtain three angles If we If we add AB we obtain AB, C, B which we reorder from smallest to largest as the first triple shown beneath each image. If we add BC we obtain A, BC, B which we reorder from smallest to largest as the second triple shown beneath each image. It is interesting to note that the second triple at all three distances from the center is 4,6,8.

From angle triples to triangles images. Table 1 from the previous section lays out the six triangles images that are obtained from these six triples. The two associated with image 7 are 2,4,12 VT and 4,6,8 VT and the two associated with image 9 are 2,6,10 VT and 4,6,8 VT. Both distances are shown in 4,6,8 VT, Figure 1.5 of the previous section. Because both lines in images 7 and 9 span an even number of vertices, the only way to have a concurrence with these angles is to use the VT type.

By contrast, the two concurrent triangles images associated with image 8 must use lines that span an odd number of vertices (**5** and **7**). Given three even angles and only odd lines, the images must be the no VT type. Figure 1.8 from the previous section shows the first image and image 10 shows the second (4,6,8 no VT) rotated so that the point of concurrency is at vertex 0.5 like 8 above with concurrency lines superimposed. Two of those lines are of length **7** and one is of length **5**. This image has 3 concurrencies because it also includes a central concurrency as noted in Table 1 of the previous section.