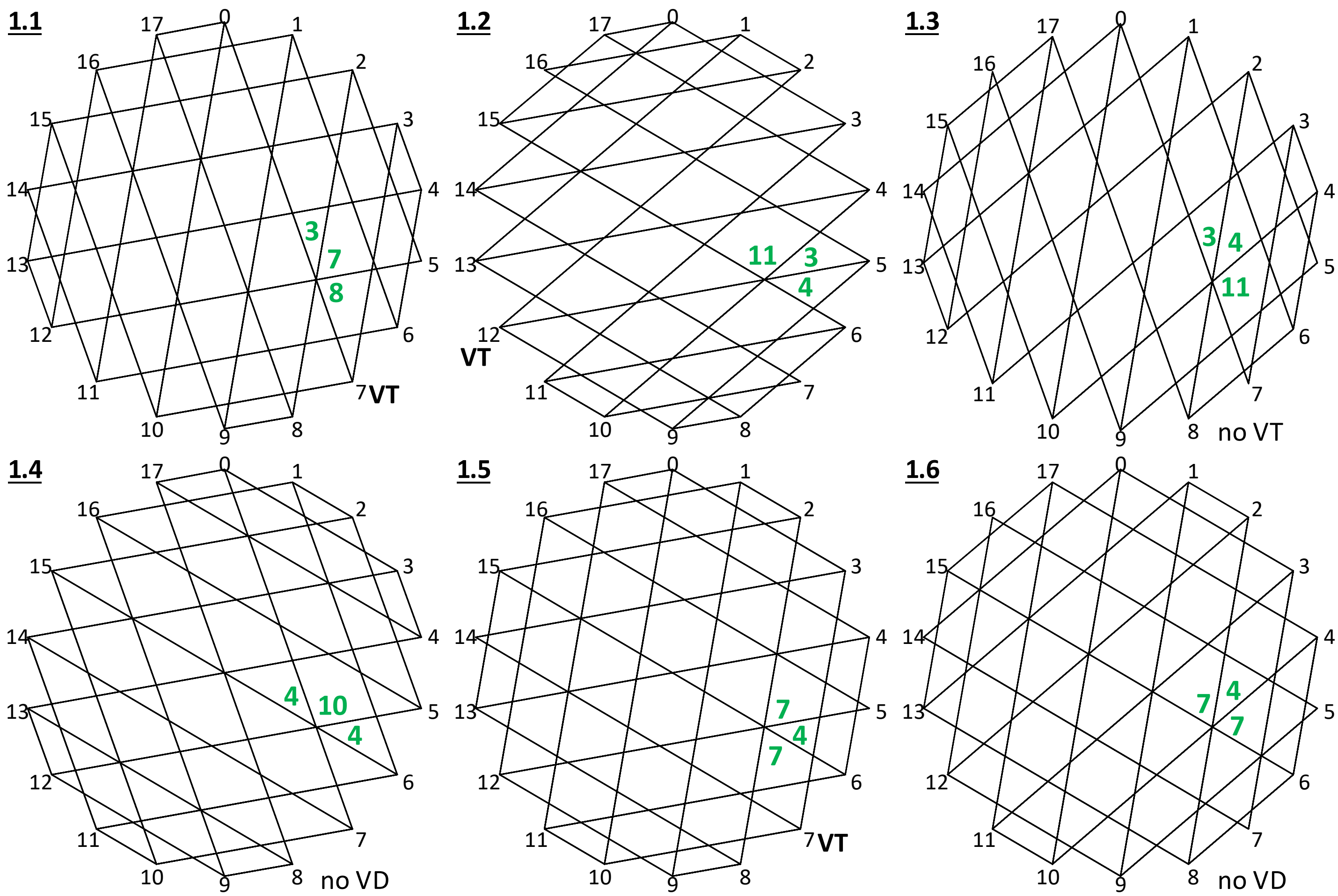
A Second Look at a 5 Lines Concurrence

In the previous section we saw why two distinct triangles images with concurrences at the same distance from the center occur when the point in question is a 4 lines point of concurrence.

In this section we examine the three 5 lines concurrences given ***n*** = 18 initially discussed in Section 7.4. Our initial focus is on the middle of those concurrences, at 0.532 of a radius, based on lines spanning 6, 7, and 9 vertices. Figure 1 images 1.2-1.6 are rotated as described in the notes to Table 1 to have the point of concurrence at the same location shown in image 1.1 and Figure 2 (much like we did in the previous section). By doing so, there can be no doubt that all six points of concurrence are the same distance from the center. Line lengths are color-coded in Figure 2 and vertices of lines in the direction opposite each angle are noted in the table with isosceles legs highlighted in gray in Table 1.

Figure 1. The ***n*** = 18 Images Created by a **5 Lines** Concurrency at 0.532 shown in Figure 2, rotational notes in Table 1.



Three of the images are VT and three are no VT. A VT label is placed near one of the VT vertices in each case (so image 1.2 has a VT at 5-9-12, (and rotational counterpart at 14-0-3) for example). **Green** angle labels are placed on each image and these angles can be seen as adding angles from Figure 2 (for degrees, multiply each angle by 10 given ***n*** = 18). This image shows the five lines passing through the common concurrence point: two span 6 vertices in shades of **red**; two span 7 in shades of **blue**; and one, the diameter, spans 9 in **purple**.

Focus on line lengths. Any concurrent image requires that ***n*** is even. Therefore, a concurrent triangles image requires three even angles or two odd and one even angle since the sum of the three angles equals ***n*** (which is even). If an image is VT, then all three lines are even, or two lines are even and one is odd since *VT image line segment spans coincide with angle spans on an even/odd basis*. This is why images 1.1, 1.2 and 1.5 are VT. No VT images switch line spans from even to odd and vice versa so that 3 even angles become 3 odd lines, like 1.4, and 2 odd angles and one even angle becomes 2 even and one odd line segments like images 1.3 and 1.6.

Focus on angles. In each 5 lines concurrence, there are 3 distinct line lengths: S, M, and L where L is the ***n***/2 diameter. The five lines are composed of two S, two M, and one L. These five lines produce three forms of angle symmetry: oppositional, and two forms of reflective.

One form of reflective symmetry is symmetry about L, the diameter line, so that if one begins listing angles starting at the diameter line (like is done at the top of Table 2 where the first angle is the intersection of L / M, the 2nd is M / S and so on), the 1st and 10th are equal, the 2nd and 9th are equal, and so on.

Whenever two lines intersect, the opposite angles created by the intersection are the same. This means the 1st and 6th are equal, the 2nd and 7th, and so on.

The second form of reflective symmetry is symmetry about the middle of the 5 angles. The 3rd and 8th angles are formed from the S / S intersection. This intersection has spans the same number of vertices on both sides of the diameter line and hence this angle MUST BE even. Call this angle C (C is 2, 4, and 2 in the three panels of Table 2).

Due to how we have enumerated angles, the 1st, 5th, 6th, and 10th angles are formed from L / M intersection and are therefore the same. Call these A. The 2nd, 4th, 7th, and 9th are formed from M / S intersection and therefore are the same. Call these B. Unlike C, there is an asymmetry in the number of vertices spanned on each side of the intersection point for A and B. This means that A and B must span at least 3 vertices.

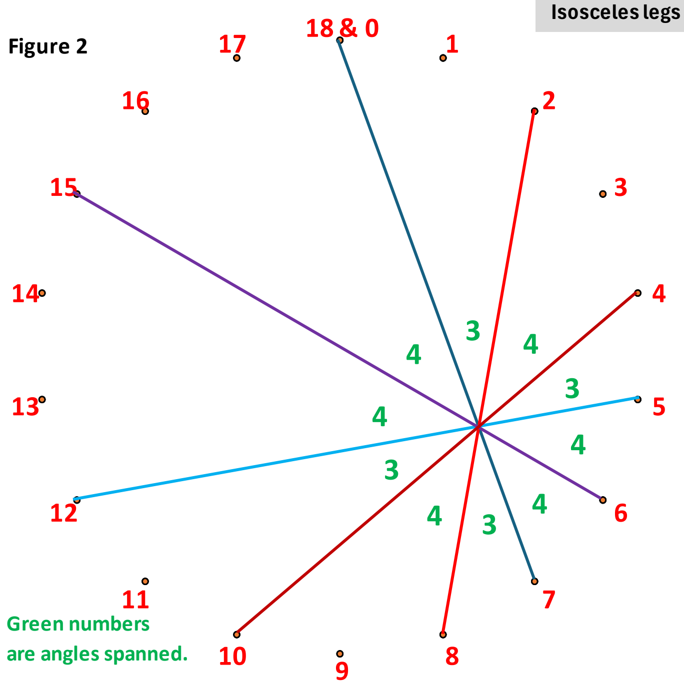
So, the 10 angles formed from a five lines concurrence form the pattern: A, B, C, B, A, A, B, C, B, A. These three values can be distinct from one another like we see with A, B, C of 5, 3, 2 or 3, 5, 2 in the left and right panel, or some may coincide like 4, 3, 4 in the middle panel of Table 2 and Figure 2.

Table 1. Rotated VT and nVT Models with Concurrence at 53.2% of the Radius*

Line opposite angle		Figure 1 Image:	1.1	1.2	1.3	1.4	1.5	1.6
		Vertices used	**3,7,8 VT**	**3,4,11 VT**	**3,4,11 nVT**	**4,4,10 nVD**	**4,7,7 VT**	**4,7,7 nVD**
a , o***a***		1	5	6	4	0	2	6
		2	12	15	10	7	8	15
b , o***b***		3	0	4	0	5	5	2
		4	7	10	7	12	12	8
c , o***c***		5	2	5	2	6	6	4
		6	8	12	8	15	15	10
even/odd spans			2o/1e VT	2o/1e VT	2e/1o nVT	3 odd nVT	2o/1e VT	2e/1o nVT

* 3,4,11 VT is rotated 5 clockwise from VT using ***J*** = 14, ***K*** = ***W*** = 17, ***V*** = 4.
3,4,11 no VT is rotated 1 clockwise from no VT using ***J*** = 7, ***K*** = ***W*** = 10, ***V*** = 4.
4,4,10 no VD is rotated 6 clockwise from no VT using ***J*** = 17, ***K*** = ***W*** = 3, ***V*** = 4.
4,7,7 VT is rotated 5 clockwise from VT using ***J*** = 17, ***K*** = ***W*** = 3, ***V*** = 7.
4,7,7 no VD is rotated 1 clockwise from no VT using ***J*** = 10, ***K*** = ***W*** = 14, ***V*** = 7.

Figure 2



Note: The line from **3** to **9** does not go through this 5 lines point of concurrence. It is perpendicular to **6-15** and that intersection point is exactly at 0.5 of the radius rather than at 0.532 of the radius.

Table 2. Angle Analysis of the Three 5 Lines Concurrence Points at Ranks 7, 6, and 3, given ***n*** = 18 (Triangles images noted beneath)

Possible ***a*** , ***b*** , ***c*** triangles – concurrence at 0.653

#	1	2	3	4	5	6	7	8	9	10
∠	5	3	2	3	5	5	3	2	3	5
	13	**8**	**3**				**13**	**10**	**5**	
	3	**8**	**13**				**2**	**5**	**10**	
2	3	5	5	3		**3**	2	3	5	5
	10	**8**	**5**			2 ***a*** = 2 start at #3				
	3	**5**	**8**			1 ***a*** = 3 start at #2				
5	3	2	3	5		1 ***a*** = 5 start at #1				

Possible ***a*** , ***b*** , ***c*** triangles – concurrence at 0.532

#	1	2	3	4	5	6	7	8	9	10
∠	4	3	4	3	4	4	3	4	3	4
	11	**8**	**4**				**10**	**7**	**3**	
	4	**7**	**11**				**4**	**7**	**11**	
3	4	3	4	4		**4**	4	3	4	3
	This is the set of angles consistent with Figures 1 and 2.									
	2 ***a*** = 3 start at #2					2 ***a*** = 4 start at #5				

Possible ***a*** , ***b*** , ***c*** triangles – concurrence at 0.347

#	1	2	3	4	5	6	7	8	9	10
∠	3	5	2	5	3	3	5	2	5	3
	11	**8**	**5**				**10**	**8**	**3**	
	5	**8**	**11**				**5**	**7**	**12**	
2	5	3	3	5		**3**	5	2	5	3
	11	**6**	**3**			2 ***a*** = 2 start at #3				
	2	**7**	**10**			3 ***a*** = 3 start at #1				
5	2	5	3	3		1 ***a*** = 5 start at #2				

Each block uses a half circle of 5 angles starting at ***a*** in bottom row. Second row is sideways sum from botttom row starting at second angle. Top row is 18-***a*** -middle.

Legend: ***a*** **,** ***b*** **,** ***c*** triples with both VT and no VT concurrence.
Shading: ***a*** | ***b*** | ***c*** | **already counted** | **isosceles**

Angle numberings starts with one side of angle #1 as the diameter of the circle having a line length of 9 = ***n*** /2. Distance ranking from Section 7.4.

Rank 7, 5 Line lengths, 5, 5, 6, 6, 9

o***a***	o***b***	o***c***	***a*** , ***b*** , ***c*** type	o***a***	o***b***	o***c***	***a*** , ***b*** , ***c*** type
6	5	5	2,3,13 VT	5	6	9	**3,5,10 VT**
5	5	9	2,8,8 noVD	6	6	5	**3,5,10 noVT**
9	6	6	**5,5,8 noVD**	6	5	9	**5,5,8 VT**

Rank 6, 5 Line lengths, 6, 6, 7, 7, 9 (Figures 1.1-1.6)

o***a***	o***b***	o***c***	***a*** , ***b*** , ***c*** type	o***a***	o***b***	o***c***	***a*** , ***b*** , ***c*** type
9	6	7	**3,4,11 VT**	7	7	9	4,4,10 noVD
6	7	6	**3,4,11 noVT**	6	7	9	**4,7,7 VT**
7	7	6	3,7,8 VT	9	6	6	**4,7,7 noVD**

Rank 3, 5 Line lengths, 7, 7, 8, 8, 9

o***a***	o***b***	o***c***	***a*** , ***b*** , ***c*** type	o***a***	o***b***	o***c***	***a*** , ***b*** , ***c*** type
8	7	7	2,5,11 VT	7	9	8	3,5,10 VT
9	7	7	2,8,8 noVD	7	9	8	3,7,8 VT
8	7	8	5,6,7 noVT	8	8	9	3,3,12 noVD

Concurrence points for stacked stars Section 7.3 images 3.1 - 3.3 analyzed at left, middle, and right with lines o***a*** , o***b*** , o***c*** opposite angles ***a*** , ***b*** , ***c*** .

Obtaining 3 angles from 5 angles. The middle portion of Table 2 shows a systematic way to search for possible triangles images with a concurrence at the point in question. The idea is to examine possible combinations of 3 angles that can be made from a set of 5 adjacent angles which form a half circle.

Triangles images require 3 angles that sum to ***n***, but we have 5 angles that sum to ***n*** (noted in the top portion of Table 2). This means that some of the adjacent angles must be combined. We can start our analysis from any location, not just at #1. The ONLY thing that starting at #1 guarantees is that one of the lines will be a diameter.

Each possible triangles image must have a smallest angle, ***a***. Given ***n*** = 18, that angle must be at most ***n***/3 = 6 and that angle must be either A, B, or C (or the sum A+B or B+C so long as this does not exceed ***n***/3). Locations of possible smallest angle ***a*** are noted for each 5 lines concurrence. For example, the smallest small angle is ***a*** = 2 starting at #3 in the Rank 7 and Rank 3 concurrences but is ***a*** = 3 starting at #2 in the Rank 7 angle # 3 in the Rank 6 concurrence. These values of ***a*** are highlighted in orange at bottom row left side of the top left block in each panel of Table 2. Possible values for ***b*** and ***c*** are obtained by adding subsequent angles in the middle row with 18-***a***-middle in the top row of each block. Values for ***b*** and ***c*** are color-coded yellow and green unless two of the angles are the same so that the result is an isosceles triangle which is shaded grey just like in Table 1. Triples that have already been obtained are highlighted in blue. Actual images containing concurrence at each rank are shown in the bottom portion of Table 2.

The angle creation rubric just described creates four distinct ***a***,***b***,***c*** triples for the left and middle panels and six distinct triples in the right panel. Visual inspection of VT and no VT versions of each triple leads to an interesting conclusion: In the left and middle panels, both VT and no VT images exhibit concurrence at this distance from the origin for two of the four triples, but none of the triples in the right panel exhibit concurrence at this distance for both types of images. All three panels have six images with concurrences at this rank.

The next section will provide a detailed reason why this is the case, but we can provide a bit of the heuristics here. We focus on the middle set of angles which are the basis for Figures 1.1-1.6. We see both types of images for 3,4,11 and 4,7,7 but only one version of 3,7,8 and 4,4,10. If you look at Figure 2 you can see why. We can obtain the angles 7 and 11 in a couple of different ways, but there is only one way to obtain an angle of 8 given the angle sequence described above. Looking at Figure 2, 8 can only occur if the diameter is NOT part of the image since adding the two 4s next to one another separated by the diameter line is the only way to add consecutive numbers to get 8 given the 10 angle sequence 4,3,4,3,4,4,3,4,3,4. Similarly, 10 can only be obtained one way, by adding 3+4+3 given these angles.