A General Lines and Angles Rubric for 5 lines Concurrence

The previous section provided a method to find triangles images that have concurrent points at the same distance from the origin using ***n*** = 18 as a point of reference. This method searched over possible ***a*** values to find values for the other two angles, ***b*** and ***c***, that could be obtained given the 5 angles pattern visible at each 5 lines point of concurrence.

For ***n*** = 18, there were three such distances and each of those distances was associated with six distinct triangles images. In two cases, there were four distinct angle triples but two of them had concurrence for both VT and no VT image types. For the third, there were six distinct angle triples and none of them exhibited concurrence for both VT and no VT modes. Individual angles are created from intersecting lines and follow these 10-line/ten-angle pattern where S, M, and L = ***n***/2 are line lengths and A, B, and C are angles: L,A,M,B,S,C,S,B,M,A,L,A,M,B,S,C,S,B,M,A as discussed in the previous section.

Finding angle triples. The general strategy for finding ***a***,***b***,***c*** triples shown in the left portion of the table is to examine a half-circle of lines and angles. Use a 5 line/5 angle sequence starting with the line before A, B, or C and fix the first two lines and hence the first angle, then systematically remove two of the last 3 lines in the sequence and replace them with + signs to create the second two angles. The resulting string tells us all three angles as well as which three line lengths will be used at the concurrence point. We need only check 6 combinations to find all possible images.

This will work for any ***n***, not just ***n*** = 18. Applying the general strategy to the three 5 lines concurrence distances found for ***n*** = 18 is shown in the middle portion of the table. It is worth pointing out that the triples angle order is reorganized so that the smallest angle is written first as is usual in our discussion of triangles images (so that triple 1 is noted as 3,5,10 for the rank 7 image even though A = 5 and B = 3 here). The order of images is different from discussed in Table 2 of the previous section but each of the images found there is seen using this general strategy.

When is a triple VT, when is it no VT? The even and odd nature of the three angles in the triangles model mimics the even and odd nature of lines at a point of concurrence for VT images. From Poonen and Rubinstein, discussed in Section 7.3, we know that all ***n*** with 5 lines concurrence must be divisible by 6 and be greater than 12. Since the three angles ***a***+***b***+***c*** = ***n***, either two must be odd and one even or all three must be even. Therefore, VT images occur when there are two odd and one even length lines or if all three are even and no VT images occur when there are two even and one odd length lines or if all three are odd.

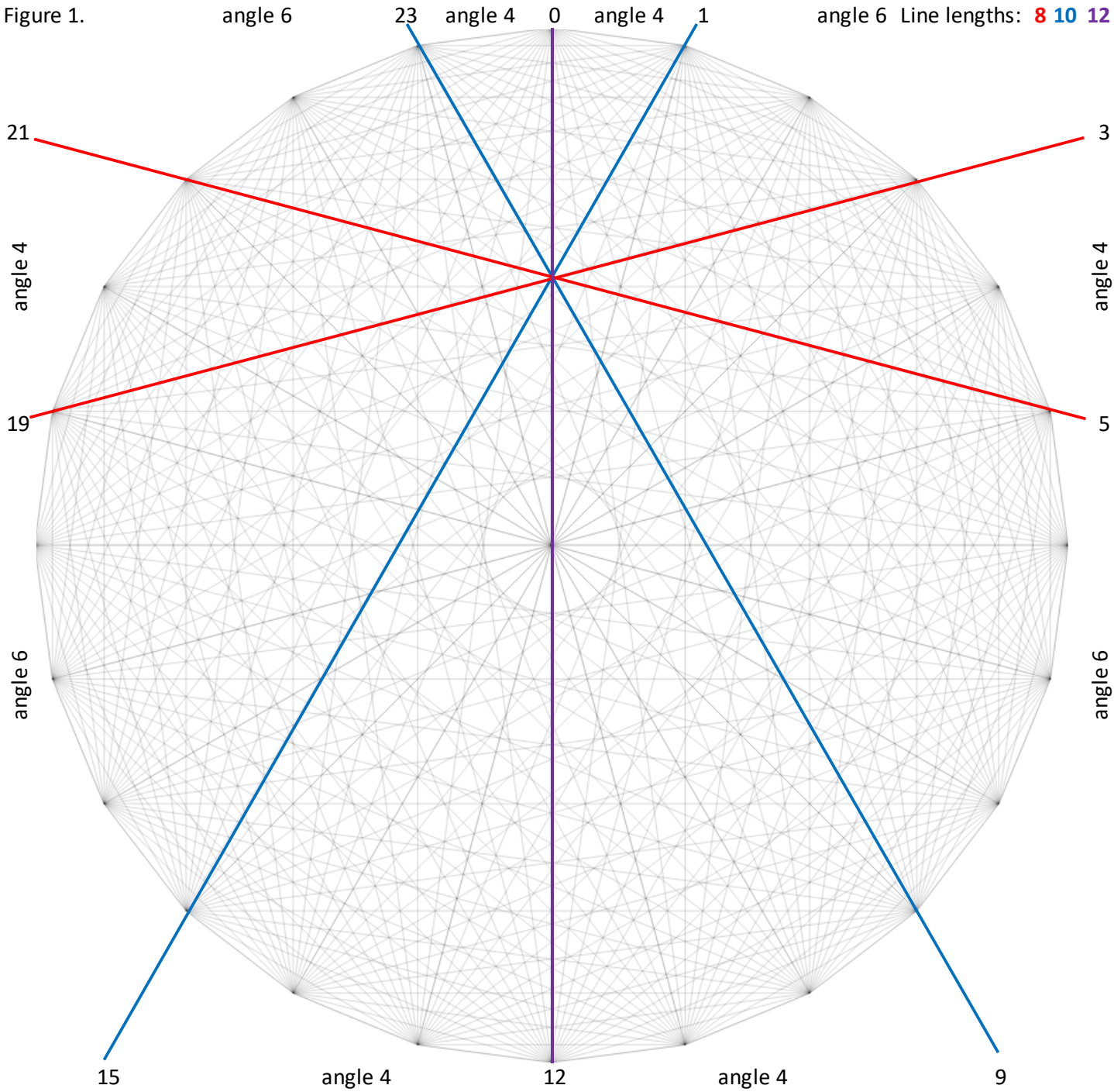
Given ***n*** = 18, L = ***n***/2 = 9 so one line is odd while one of M and S is even and the other is odd. This creates both VT and no VT type images. Interestingly, triples 1, 2, 3, and 6 produce VT or no VT for all three points, but triples 4 and 5 have both types because each use only S and M sized lines and the even/odd nature of S and M varies across points.

Whether there is angular redundancy in the triangles images or not depends on how the angles A,B,C are organized as can be seen by comparing rank 7 (5,3,2) and rank 3 (3,5,2) versions of A,B,C.

General Strategy for Creating Triangles Images from 5 Lines Points of Concurrence^																		0.653 rank 7 images							0.532 rank 6 images							0.347 rank 3 images							5 lines for ***n*** = 24*						
																		Angles			Lines				Angles			Lines				Angles			Lines				Angles			Lines			
Angle #			1		2		3		4		5		6		7			A	B	C	S	M	L		A	B	C	S	M	L		A	B	C	S	M	L		A	B	C	S	M	L	
Line/Angle	L	A	M	B	S	C	S	B	M	A	L	A	M	B				5	3	2	5	6	9		4	3	4	6	7	9		3	5	2	7	8	9		4	6	4	8	10	12	
Triple #									Start at #		Lines Used							#	***a***	***b***	***c***	Type			#	***a***	***b***	***c***	Type			#	***a***	***b***	***c***	Type			#	***a***	***b***	***c***	Figure		
1	L	A	M	B	S	C	+	B	+	A		1		L	M	S		1	**3**	**5**	**10**	**VT**			1	3	**4**	**11**	**VT**			1	3	5	10	VT			1	4	6	14	VT	2A	
2	L	A	M	B	+	C	S	B	+	A		1		L	M	S		2	**5**	**5**	**8**	**VT**			2	**4**	**7**	**7**	**VT**			2	3	7	8	VT			2	4	10	10	VT	2B	
3	L	A	M	B	+	C	+	B	M	A		1		L	M	M		3	**5**	**5**	**8**	**no VT**			3	4	4	10	no VT			3	3	3	12	no VT			3	4	4	16	VT	2C	
4	M	B	S	C	S	B	+	A	+	A		2		M	S	S		4	2	3	13	VT			4	**3**	**4**	**11**	**no VT**			4	2	5	11	VT			4	4	6	14	VT	2A	
5	M	B	S	C	+	B	M	A	+	A		2		M	S	M		5	**3**	**5**	**10**	**no VT**			5	3	7	8	VT			5	5	7	6	no VT			5	6	8	10	VT	2D	
6	S	C	S	B	+	A	L	A	+	B		3		S	S	L		6	2	8	8	no VT			6	**4**	**7**	**7**	**no VT**			6	2	8	8	no VT			6	4	10	10	VT	2B	
	M	B	S	C	+	B	+	A	L	A	duplicates 1							Each set of triples is created using the general pattern at left.																															
	S	C	S	B	M	A	+	A	+	B	duplicates 4							Although these three sets of ***n*** = 18 images are in a diiferent																															
	S	C	S	B	+	A	+	A	M	B	duplicates 4							order from that presented in Table 2 of the last section, the																															
^Create 3 angles by replacing 2 of the last three lines in a 5 Line/Angle sequence with +																		same six triangles images result at each distance.																															

*Each is 0.518 from the center same as ***n*** = 12, 2,3,7 no VT and 3,4,5 VT from Section 7.5.

The right portion of the table shows ***n*** = 24. Interestingly, even though ***n*** is larger, there is only one 5 lines point of concurrence (per wedge, or 24 total) and it is 0.518 from the center (just like Section 7.2, images 3 and 7). The 23·24/2 = 276 line ***n*** = 24 Figure 1 focuses on the vertex 0 5 lines point of concurrence. It has a total of 7,297 total intersections (according to Table 7 of Poonen and Rubinstein). Five lines are overlaid on the image, two of length **8**, two of length **10** and the vertical diameter of length **12**. Vertices where the lines end and angles created by these concurrent lines are noted around the edge. If you follow each line you will see that each a part of TWO 5 lines points of concurrence. The **red lines** are also part of the vertex 2 and 22 = ***n***-2 5 lines point of concurrence. The **blue lines** are also part of the vertex 8 and 16 = ***n***-8 5 lines point of concurrence. The **purple** vertical diameter is on the vertex 12 5 lines point of concurrence.



The 6 triples created by the general strategy for creating triangles images noted above produce only 4 distinct images because A = C so that 1 and 4 have the same angle triples as do 2 and 6. Unlike when this happened with the ***n*** = 18 rank 6 image that was the focus of the start of the previous section (and the 0.532 rank 6 point in the middle part of the table in this section), all three lines are even (**8**, **10** and **12**) and all angles are even (since A, B, and C are even) hence all

concurrent images at this distance must be VT. These four images are shown as Figure 2A-2D. Each image highlights the concurrence at 0.518 from the center via dots. Figures 2C and 2D are self-explanatory so we now focus on 2A and 2B.

Although table rows 1 and 4 produce a result of 4,6,14 VT shown in 2A, they differ in the lines used to attain these angles. The two points noted in **blue** use all three line sizes as prescribed by 1, but the two noted in **red** use two small and one medium line size as prescribed by 4.

Table rows 2 and 6 produce 4,10,10 VT shown in 2B. The two **red** points are part of diameter concurrence with two small and one large lines as prescribed by 6 (in 6, angle 4 = C is created using the two small lines (S,C,S …) so the diameter line helps create the base angles of the isosceles image). By contrast, the four **blue** points use all three line sizes as prescribed by 2 (in 2, angle 4 = A is created using the large and medium lines (L,A,M …) so the diameter line helps create the apex angle). In the context of Table 2 section 7.3, the **red** are 2 of the 9 C D points and the 4 **blue** are the 2Sp points.