A 30-Gon with all its Diagonals

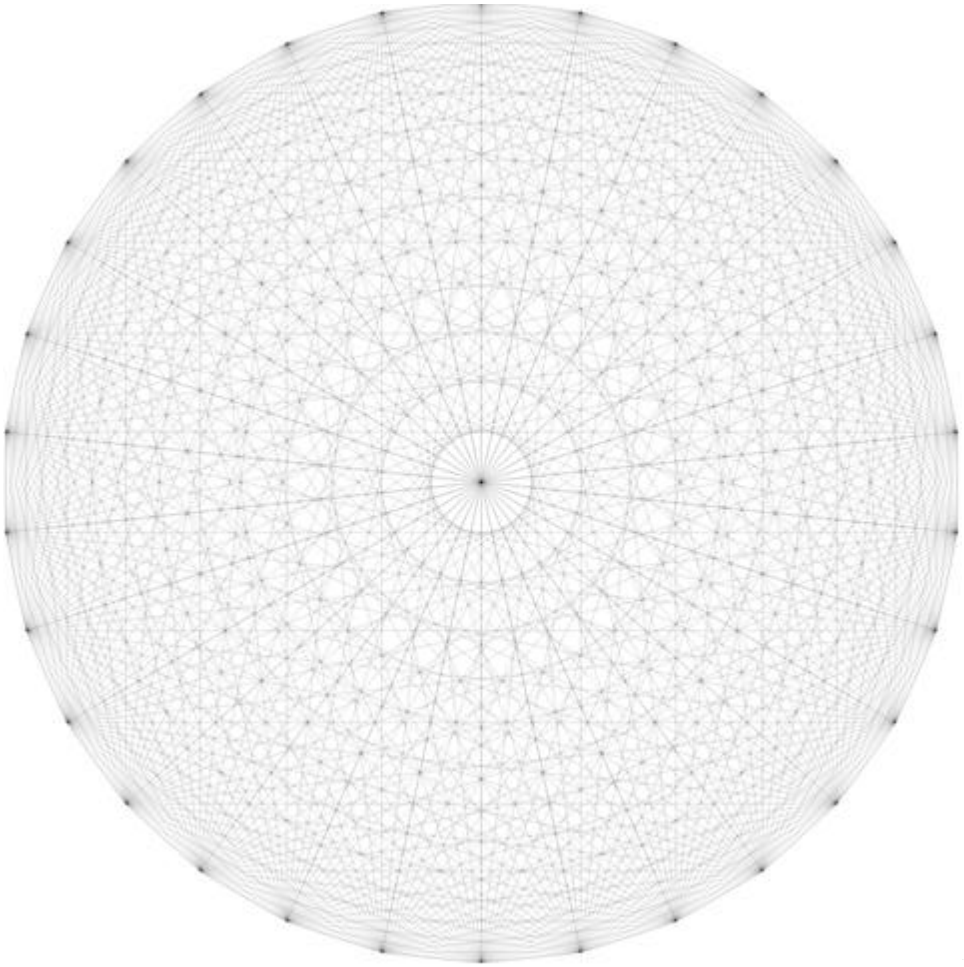
Image **I** at bottom left reproduces Figure 1 from Poonen and Rubinstein. The 435 = 30·29/2 line image has 16,801 interior intersection points of which: 13,800 are 2 lines; 2,250 are 3 lines; 420 are **4 lines**; 180 are **5 lines**; 120 are **6 lines**; 30 are 7 lines, and 1 is 15 lines. 30 is the smallest ***n*** for which 7 regular ***n***-gon diagonals meet at a point. One stunning result from Poonen and Rubinstein is that one can never have more than 7 diagonals meet at a non-central point.

Image **II** is the 360/***n*** = 12° wedge with 420/30 = 14 **4 lines**, 180/30 = 6 **5 lines**, and 120/30 = 4 **6 lines** concurrences circled. The left side is lined up on the ray from the center to midway between vertices 29 and 0 but the right side shows that midway set of intersections to more easily identify concurrences on the ray that is midway between vertices 0 and 1 at 6° from vertical. The 7 lines concurrence in this wedge is on the positive vertical radius between star sizes 8 and 9 (star sizes are noted at the right).

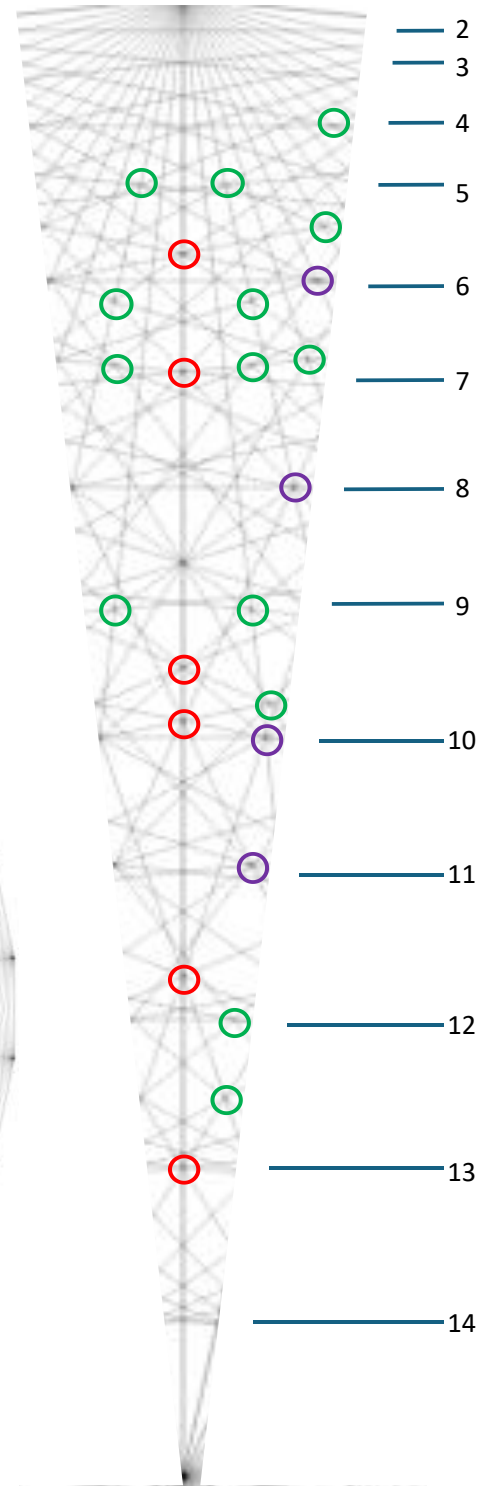
Image **I** is an intricate weave but there are distinct levels that look like stacked circles. These are created by different jump stars. (It is worth noting that they only look like circles, in fact they are ***n***-gons, here 30-gons.)

The most obvious ones are the inner levels, like the one that is closest to the center which is created by a 30,14-star. A careful inspection shows that even jumps ***J*** = 2***k*** are horizontal lines (connecting vertices ***k*** with 30-***k***) while odd jumps are pairs of intersecting lines along the vertical axis with the smallest sized interior angle possible, 2·180/30 = 12° since the lines are separated from one another by a single vertex on either side of the vertical centerline. For example, the star size 3 intersection shown in **II** intersects lines from 29-2 and 28-1 but the star size 5 intersection intersects lines 28-3 with 27-2.

I. A 30-gon with all its diagonals.



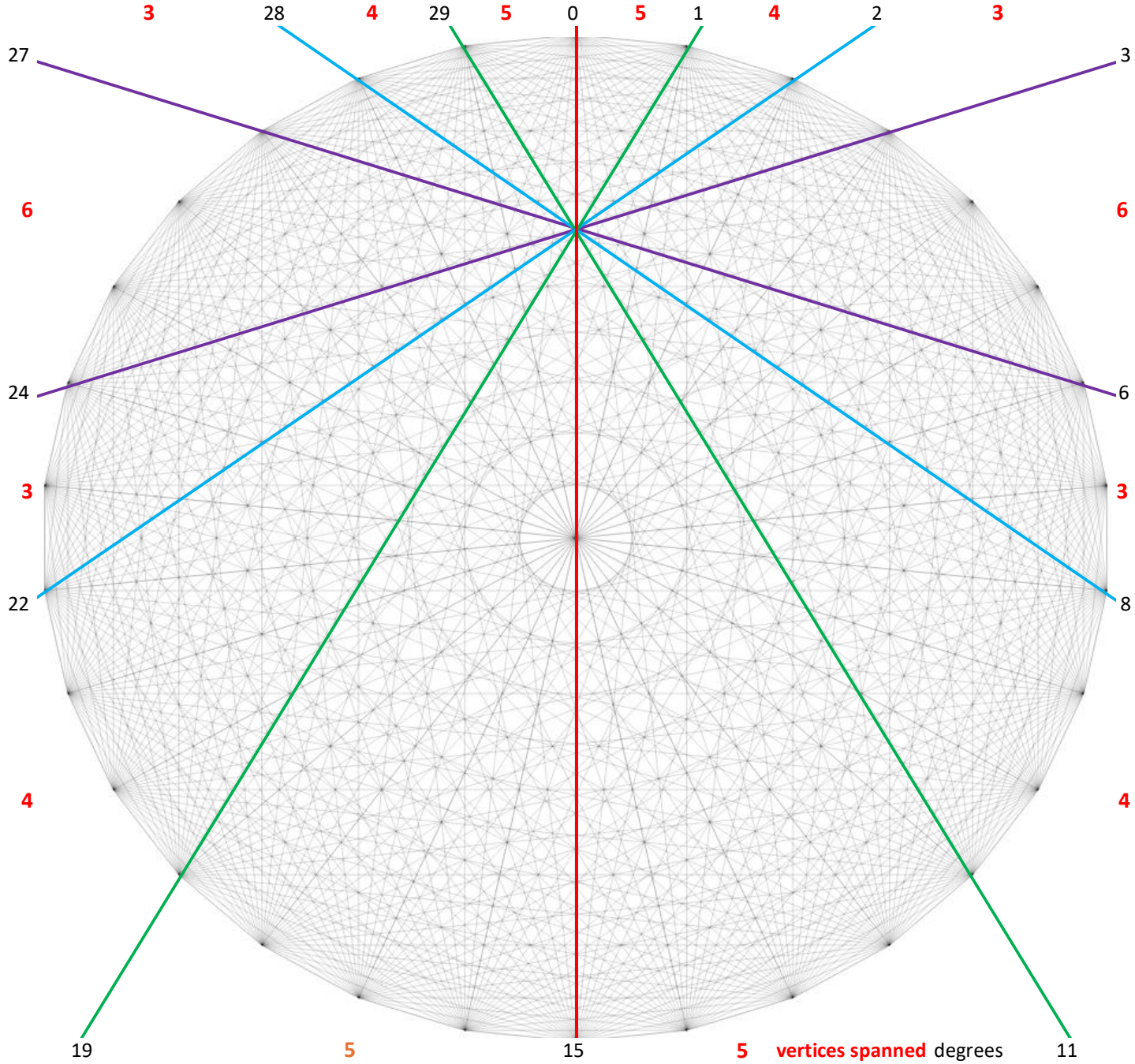
II. 12° Wedge at Vertex 0 Star sizes



Interestingly, the angles created at the 7-lines concurrence points are not uniform. As image **III** shows, the angles vary from top to bottom by 30°, 24°, 18°, 36°, 18°, 24°, and 30° (or **5**, **4**, **3**, **6**, **3**, **4**, and **5** vertices spanned) according to the interior angle theorem. For convenience, the perimeter of the image has vertices numbered for each colored line and angles (in number of vertices spanned) in **red.** The length of lines creating this 7-lines concurrence point span four sizes. The colored overlays show two lines each spanning **9**, **10**, and **12** vertices and the vertical diameter spans **15** vertices. Note how the pairs of lines appear tangent to the stacked circles at levels **12** (3rd circle from the center), **10**, and **9**.

In the next section we will use image **III** and the 5-lines rubric created in the last section to see how many triangles images having points of concurrence at this distance from the center can be created from a 7-lines point of concurrence such as this ***n*** = 30 example.

III. The Seven Diagonals Concidence Point Beneath Vertex 0 in a 30-gon between Levels 8 and 9 at 0.618 of the Radius



The 7 lines span **9**, **10**, **12** & **15** vertices of the 30-gon.

Note: The **green lines** are part of a 30,12-star.
The **blue lines** are part of a 30,10-star.
The **purple lines** are part of a 30,9-star.

Angles created by:		vertices spanned	degrees
red to **green**	**5**		30 °
green to **blue**	**4**		24 °
blue to **purple**	**3**		18 °
purple to **purple**	**6**		36 °