A General Lines and Angles Rubric for 7-lines Concurrence

There is one more pair of lines in the 7-lines concurrence (now S, M, L, and d for diameter) and 4 angles rather than 3, but rest of the rules and patterns seen in the 5-lines rubric presented two sections ago apply. These are noted at the bottom of the table and will not be explained at length here.

Table header columns: **end** | **1st half circle, lines and angles** / **Partial 2nd half** (pattern columns: L A d A L B M C S D S C M B L A d A L B M C) | **Duplicates (First in blue)** | **#** | **First** | **Second** | **Third** | **Lines used** | **n = 30 Angles** (Type, 1st, 2nd, 3rd) | **Sorted** (a, b, c) | **Image**

1st half / Partial 2nd half pattern	Dup	#	First	Second	Third	Lines used	Type	1st	2nd	3rd	a	b	c	Image
d A L B M C + D + C + B + A		1	A	B	C+D+C+B+A	d L M	no VT	5	4	21	4	5	21	
d A L B + C S D + C + B + A		2	A	B+C	D+C+B+A	d L S	VT	5	7	18	5	7	18	
d A L B + C + D S C + B + A		3	A	B+C+D	C+B+A	d L S	VT	5	13	12	5	12	13	
d A L B + C + D + C M B + A		4	A	B+C+D+C	B+A	d L M	no VT	5	16	9	5	9	16	
d A L B M C + D + C + B L A		5	A	B+C+D+C+B	A	d L L	no VT	5	20	5	5	5	20	1
L B M C S D + C + B + A + A		6	B	C	D+C+B+A+A	L M S	no VT	4	3	23	3	4	23	4
L B M C + D S C + B + A + A		7	B	C+D	C+B+A+A	L M S	no VT	4	9	17	4	9	17	
L B M C + D + C M B + A + A		8	B	C+D+C	B+A+A	L M M	VT	4	12	14	4	12	14	
L B M C + D + C + B L A + A		9	B	C+D+C+B	A+A	L M L	VT	4	16	10	4	10	16	7
L B M C + D + C + B + A d A	1	10	B	C+D+C+B+A	A	L M d	no VT	4	21	5				
M C S D S C + B + A + A + B		11	C	D	C+B+A+A+B	M S S	VT	3	6	21	3	6	21	
M C S D + C M B + A + A + B		12	C	D+C	B+A+A+B	M S M	no VT	3	9	18	3	9	18	8
M C S D + C + B L A + A + B		13	C	D+C+B	A+A+B	M S L	no VT	3	13	14	3	13	14	
M C S D + C + B + A d A + B		14	C	D+C+B+A	A+B	M S d	VT	3	18	9	3	9	18	5
M C S D + C + B + A + A L B	6	15	C	D+C+B+A+A	B	M S L	no VT	3	23	4				
S D S C M B + A + A + B + C	11	16	D	C	B+A+A+B+C	S S M	VT	6	3	21				
S D S C + B L A + A + B + C		17	D	C+B	A+A+B+C	S S L	VT	6	7	17	6	7	17	
S D S C + B + A d A + B + C		18	D	C+B+A	A+B+C	S S d	no VT	6	12	12	6	12	12	2
S D S C + B + A + A L B + C	17	19	D	C+B+A+A	B+C	S S L	VT	6	17	7				
S D S C + B + A + A + B M C	11	20	D	C+B+A+A+B	C	S S M	VT	6	21	3				
d A + B M C S D + C + B + A	14	21	A+B	C	D+C+B+A	d M S	VT	9	3	18				
d A + B M C S D S C + B + A		22	A+B	C+D	C+B+A	d M S	VT	9	9	12	9	9	12	6
d A + B M C S D + C M B + A		23	A+B	C+D+C	B+A	d M M	no VT	9	12	9	9	9	12	3
d A + B M C S D + C + B L A	4	24	A+B	C+D+C+B	A	d M L	no VT	9	16	5				
L B + C S D S C + B + A + A	17	25	B+C	D	C+B+A+A	L S S	VT	7	6	17				
L B + C S D + C M B + A + A		26	B+C	D+C	B+A+A	L S M	no VT	7	9	14	7	9	14	
L B + C S D + C + B L A + A		27	B+C	D+C+B	A+A	L S L	no VT	7	13	10	7	10	13	9
L B + C S D + C + B + A d A	2	28	B+C	D+C+B+A	A	L S d	VT	7	18	5				
M C + D S C M B + A + A + B	12	29	C+D	C	B+A+A+B	M S M	no VT	9	3	18				
M C + D S C + B L A + A + B	26	30	C+D	C+B	A+A+B	M S L	no VT	9	7	14				
M C + D S C + B + A d A + B	22	31	C+D	C+B+A	A+B	M S d	VT	9	12	9				
M C + D S C + B + A + A L B	7	32	C+D	C+B+A+A	B	M S L	no VT	9	17	4				
L A + A L B M C + D + C + B	9	33	A+A	B	C+D+C+B	L L M	VT	10	4	16				
L A + A L B + C S D + C + B	27	34	A+A	B+C	D+C+B	L L S	no VT	10	7	13				
L A + A L B + C + D S C + B	27	35	A+A	B+C+D	C+B	L L S	no VT	10	13	7				
L A + A L B + C + D + C M B	9	36	A+A	B+C+D+C	B	L L M	VT	10	16	4				

Each row starts with a line size (S, M, L, d) and is a combination of creating 3 angles from 7 with first angle no larger than $n/3$. To create 3 angles, four lines are replaced with + signs so adjacent angles are added. For $n = 30$, line lengths are S = 9, M = 10, L = 12 and the diameter, d = 15, with angles starting at the diameter line of A = 5. B = 4, C = 3 and D = 6. The same three symmetries exist here as with 5 lines concurrence: oppositional symmetry; symmetry about diameter d; and symmetry about D, the angle created using the two smallest lines S. Some of the 36 possibilities duplicate others (number and first angle of the duplicate noted in **blue**). In the end, 19 triples by type are possible, two of which are available in both VT and no VT versions (noted in **red**). Isosceles legs are highlighted gray. There is only one way to sum to 10 so the equilateral triangles image 10,10,10 has no concurrence points at 0.618 from the center. The final column notes nine images shown including all isosceles images and triples with VT and no VT versions.

Turning 7 angles into 3 means replacing 4 lines with + signs in the line/angle/line/… pattern laid out at the top of the left hand side of the table. By looking closely, you could have reduced the number of possibilities checked since duplicates are reasonably easy to identify. Despite this, the choice was made to show the entire pattern: each single 1st angle (A, B,

C, D) has 5 ways to place 4 + signs (so 1-20 are shown); each double 1st angle (A+B, B+C, C+D, A+A) has 4 ways to place 3 + signs (so 21-36 are shown). In the end, 19 unique triangles images emerge for **n** = 30 that have concurrences using 3 of the 7 lines (of lengths 9, 10, 12, and 15) and 4 angles (A = 5, B = 4, C = 3, and D = 6) noted in image **III** from the last section at a distance of 0.618 of the **n**-gon radius from the center. The right hand side of the table identifies 17 angle triples that have concurrences at this distance, 15 of which are either VT or no VT but NOT both, and 2 of which (noted in **red**) are BOTH.

Nine of these images are shown below (and are noted in the right column of the table). Each includes a total concurrences in **blue**, total triangles count in **red** and **label** at lower right, and a **red circle** at 0.618 so that it is easy to see the commonality of distance from the origin in each image. The top row shows the no VT isosceles images with diameter noted at the bottom left of each image (with **b**-1 total concurrences). Images **3** and **6**, and **5** and **8**, are paired versions of the two triples with both VT and no VT styles having concurrence at this distance. Note that the non-diameter isosceles image **6** has 4 concurrences at this distance and two more outside the circle at 0.874 near vertices 7 and 22.