

## An Introduction to Two Lines Images

By removing one of the three directions in triangles images, one obtains quadrilaterals with opposing parallel sides rather than triangles images. These come in two obvious forms, rectangles and parallelograms depending on whether adjacent angles are the same or differ from one another. Since opposing sides are parallel, two adjacent angles are supplementary to one another (meaning they sum to  $180^\circ$ ). When they are the same, the images are rectangles and when they differ, they are parallelograms (although a rectangle is formally a parallelogram).

For esthetic purposes, it is useful to treat rectangles and parallelograms differently, much in the same way we treated isosceles and scalene triangles images differently. Here are standard forms for both using the *General Triangles* file.

**A standard form for rectangles.** These equations produce horizontal and vertical lines, and lines that are slightly downward sloping and steeply upward sloping in the two versions. This can be accomplished with a single parameter that takes on two values, 0 and 1. Call this parameter **no VT** and put it in the green unprotected part of the *General Triangles* file. [We use **no VT** despite the image having only quadrangles in order to be consistent with earlier usage, especially [here](#).] Right angles require even  $n$ . Set  $J = \text{no VT}$ ,  $K = J$ ,  $V = J$ ,  $W = n/2$ . When **no VT** = 0 the image has horizontal and vertical lines, and when **no VT** = 1, the downward sloping lines are parallel to the line from 0 to 1 and these lines are perpendicular to the steeply upward sloping lines that are parallel to the line from 1 to  $n/2$ .

The first thing to notice is that  $n = 4k$  has two distinct images because **no VT** = 0 creates even horizontal and vertical lines (the smallest line at top is from 1 to  $n-1$  and at bottom is from  $2k-1$  to  $2k+1$  with right from  $k-1$  to  $k+1$  and left from  $3k-1$  to  $3k+1$ ), but **no VT** = 1 creates odd lines (the smallest line at top is from 0 to 1 and at bottom from  $2k$  to  $2k+1$  with right from  $k$  to  $k+1$  and left from  $3k$  to  $3k+1$ ). By contrast,  $n = 4k+2$  has one distinct image because **no VT** = 0 creates even horizontal lines and odd vertical lines (the smallest line at top is from 1 to  $n-1$  and at bottom from  $2k$  to  $2k+2$  with right from  $k$  to  $k+1$  and left from  $3k+1$  to  $3k+2$ ). The  $4k+2$  **no VT** = 1 images switch evens and odds (the smallest line at top is from 0 to 1 and at bottom from  $2k+1$  to  $2k+2$  with right from  $k$  to  $k+2$  and left from  $3k+1$  to  $3k+3$ ). Therefore, for  $n = 4k+2$ , the only difference is rotational.

**A standard form for parallelograms.** These equations produce downward pointing smallest angles at 0. Label the **no VT** variable and *middle* the middle-sized angle in the *General Triangles* file (which turns out to be  $b$ ). To accomplish this, set  $J = \text{INT}((n - \text{middle})/2) + \text{no VT}$ ,  $K = J + \text{middle}$ ,  $V = 0$ ,  $W = K$ . This produces the smaller angle of the parallelogram close to centered down (and up) with the larger supplementary angles to right and left. The two non-zero angles are  $b$  and  $c$ , with  $a = 0$  (since the triangles direction  $VW$  coincides with  $OK$ ) and  $b = \text{middle}$  with  $b+c = n$ .

**Side lengths.** Whether we are analyzing rectangles or parallelograms we can further distinguish parts of the images based on having adjacent sides the same length. When this happens with rectangles, we obtain a square, and when this happens with a parallelogram, we obtain a rhombus. An interesting question is: How many squares or rhombi are in an image? Indeed, one can also examine whether there are ANY squares or rhombi in a given rectangles or parallelograms image.