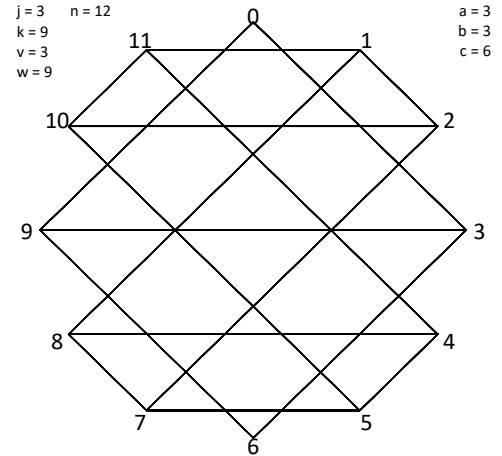


An Introduction to the *General Triangles Excel File*

The *General Triangles* file allows you to create triangular images based on any rule you wish to explore. This file allows you to choose three directions to draw families of parallel lines based on vertices of an n -gon and explore the image that results; it is a very open-ended file that allows for broad exploration. The more technical details of this file are examined in Chapter 6, but it is useful to introduce the basic aspects of this file at the start of PART I.

There are dedicated *Excel* files created to examine sharpest isosceles triangles on odd polygons and sharpest right triangles on even polygons. One additional file examines sharpest isosceles triangles on even polygons. But one can readily define other types of images beyond these specific image types. The only limiting feature is your ability to describe the attributes of the image sequences you create (and if words do not work, equations describing the parameters will suffice).

An Example. Suppose that you want to examine **isosceles right triangles** with two vertex triangles (0-3-9 and 6-9-3). This requires $n = 4K$ because we need right angles and 45° angles. In such a situation, it makes sense to start with a 12-gon since a clockface is easy to visualize. We could have hypotenuse as a vertical line (from 0 to 6) or horizontal line (from 3 to 9) but one could also imagine other images with largest hypotenuse from K to $K+6$ for $K = 1, 2, 4$, or 5. Each of these images would only differ rotationally so we are going to focus on the one with a horizontal hypotenuse, or the triangle 0-3-9. The resulting image is shown with the dashboard that created the image. The dashboard shows 4 cells, **J**, **K**, **V**, and **W** that are controlled by entering numbers or via equation together with a fifth, **n**, that can also be controlled by up/down arrows. There are 5 click-boxes together with notes beneath providing additional information. (Setting $J = 8$, $K = W = 2$, and $V = 9$ produces isosceles right triangles with no vertex triangles.)



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Vertex #'s ☒ **Points** For simplicity the first two lines use vertex 0 = (0, 1).
Circle ☐ Line 1 is 0 to $J = 9$ $J = 1, \dots, n-1$. Line 2 is 0 to $K = 3$ $K \neq J, K = 1, \dots, n-1$.
SHOW Lines ☒ Line 3: 1st vertex, $V = 9$ $V = 0, \dots, n-1$. 2nd vertex, $W = 3$ $W = 0, \dots, n-1$.

Parameter values ☒ **Notes:** Line 3 need not include J or K , although that is fine. Set $W = V = 0$ for horizontal.

a, b, c are arcs of circle summing to n & represent angles $180a/n^\circ$, $180b/n^\circ$, and $180c/n^\circ$.

Steps to obtain $0 = d$, The line VW is parallel to $0d$ with $d = \text{MOD}(W+V, n)$
 a, b , and c from $3 = g = \min(|J-K|, |J-d|, |d-K|)$ Note: h and q are unordered.
 J, K, V, W , & n : $6 = h = \text{MAX}(|J-K|, |J-d|, |d-K|) - g$ $3 = q = n - g - h$

$$3 = a = \min(g, h, q)$$

$$3 = b = n - a - c$$

$$6 = c = \text{MAX}(g, h, q)$$

With $a \leq b \leq c$ and $a+b+c = n$.

Additional information:

You can manually enter numbers in the yellow cells, or you can insert equations in those cells.

The yellow cells have been labeled so you can refer to them by name. For example entering, $= \text{INT}(n/3)$ in Q3, $= n-J$ in V3,

To rotate image $\curvearrowright$ 1 in R4, & $= n-V$ in W4, produces "near equilateral" isosceles Δ s. These are exact when n is divisible by 3.

$0 = r$, rotation factor $r = 0, 1, \dots, n-1$ using these equations

Lines= $\text{IF}(\text{MOD}(n,2)=1, 3(n-1)/2, 3n/2-3+\text{MOD}(J,2)+\text{MOD}(K,2)+\text{MOD}(V+W,2))$

$J = \text{MOD}(J_0 + 2r, n)$, $K = \text{MOD}(K_0 + 2r, n)$, $V = V_0$, $W = \text{MOD}(W_0 + 2r, n)$

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To create parallelograms, set $V = 0$ and $W = J$.

The angles return every fourth n if we tie the parameters to n using equations such as $K = \text{INT}(n/4)$, $J = 3*K$, $V = J$, $W = K$. These images show $n = 16, 20, 24$, and 28 and were obtained by unclicking *Vertex #'s* and *Parameter values* due to size.

