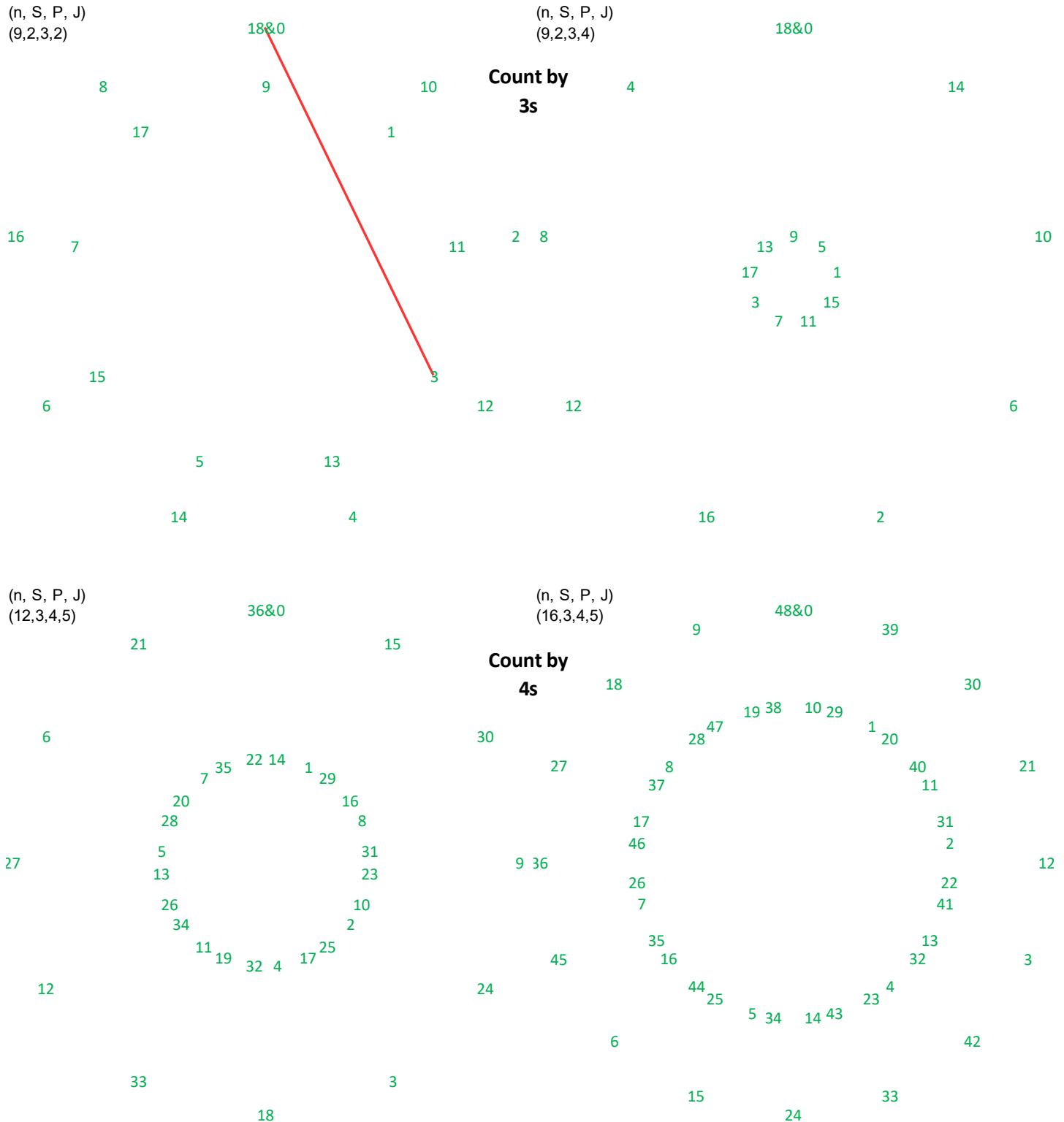


Skip Counting (Counting by P) Exercises

These pencil and ruler exercises will help reinforce the meaning of SCF (subdivision common factor). In purely geometric terms, SCF is the portion of subdivisions used in the final image. Each line on the n,J -star vertex frame has S subdivisions.

For each image, skip count by P and draw a line, then draw the second from P to $2P$ and so on to complete the image. Draw each line using the middle of the number as the subdivision endpoint. The first image shows the first line drawn in **red**. The second line will be drawn to **6**, and so on. The top row images count by 3s, the bottom row images count by 4s.

These four images have $SCF = GCD(n \cdot S / GCD(n, J), P) = P$, where GCD is the greatest common divisor function. Therefore, the final image uses only skip-counted numbers.



When SCF < P. It need not be the case that SCF = P. The next two examples use counting by 10s to show how to complete an image. For each, a table is provided to the right because counting by 10s “changes” once you have counted all the subdivisions once. The same conclusions occur when we are not counting by 10s but it is easy to count by 10s.

Upper Image. As noted at the top of the image (75&0), there are 75 = 25·3 subdivisions in the image. Counting by 10s is simple for the first 7 lines, but the 8th line is 10 subdivisions after the 7th, and there are only subdivisions numbered up to 75 so that the last 5 subdivisions counted to find the end of the 8th line must be counted on the second time around. The 8th line ends at 5, the 9th at 15, and so on until the 15th and final line ends at 75 (&0). There are 15 lines in the image because 15 = n·S/SCF = 75/5. All **used** subdivisions are multiples of 5 = SCF.

To complete the image, each subdivision is counted twice because SCF = 5. In mathematical terms, SCF = GCD(75, 10) = 5.

Lower Image. There are 36 subdivisions, so the first pass ends after 3 lines at 30, and the 4th line ends at subdivision 4 (10 = 36-30 + 4). The 7th line ends at 34 so the 8th ends at 8 (10 = 36-34 + 8) and the 10th ends at 28. The 11th ends at 2 (10 = 36-28 + 2). The 14th line ends at 32 so the 15th ends at 6 (10 = 36-32 + 6). The 18th line ends at 36 (&0).

There are 18 lines in the image because 18 = n·S/SCF = 36/2 and all **used** subdivisions are even because SCF = 2.

A General Rule. The number of times you have to count around the subdivisions to complete the image is P/SCF. It is easy to see that that is the case when SCF = 1 because P-1 subdivisions are jumped over before the first line is drawn. Since all subdivisions are used when SCF = 1, they must be used in later passes around the vertex frame.

2 NOTES: Check your work by scrolling to (n,S,P,J) for each image. Although this has not been formally introduced, this analysis is based on modular (remainder) arithmetic.

